

Stacking Sequence Optimization of Laminated Composite Plate using GA and ACO

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Abstract - Laminated composites have high specific strength and stiffness and so they are widely used in the applications in which weight reduction is critical. The structural behaviour of the composites can be further improved by optimally varying the stacking sequences of the composite plate while the weight is kept constant. Many researchers optimized the stacking sequences of the composite structure using conventional layer angles (0° , $\pm 45^\circ$, 90°). However the design space can be further increased with the choice of reduced ply angle interval. When the design space is increased, it is essential to implement the suitable optimization technique in order to find the optimum designs from the huge design space. In this paper, two most successful optimization techniques, Genetic Algorithm (GA) and Ant Colony Optimization (ACO) are proposed and compared to find the optimum stacking sequence of laminated composite plate. The results show that GA works better than ACO.

Keywords – Stacking Sequence Optimization, Genetic Algorithm, Simulated Annealing.

1. INTRODUCTION

FRP composites have high specific strength and high stiffness and so they are highly used for the applications like air craft structures and wind turbines where weight reduction is essential. The structural performances of the laminated composite s can be further improved without adding the weight by just optimizing the material system itself such as fiber orientation and stacking sequence. The composites are usually designed as thin plates and subject to the in-plane loading. This will lead to failure due to buckling. Therefore many researchers have chosen the buckling strength as the design objective while weight and in-plane strength are the design constraints. The ply orientations and stacking sequences are taken as the design variables. Le Riche et al. (1993), Soremekun et al. (2001) and Aymeriah et al. (2008) optimized the simply supported rectangular composite plate to maximize the buckling strength. The buckling strength of the stacking sequence configuration was computed analytically using CLPT theory. The plate was assumed symmetric and balanced so that 3/4th of the design variables were reduced. The plies of same orientation were grouped together in the optimum stacking sequences as they used the conventional layer angles (0° , $\pm 45^\circ$, 90°) as the design variables. Rama Mohan Rao et al. (2005) maximized the buckling strength of the symmetric rectangular composite plates subject to in-plane compressive loading. They also used conventional ply angles as design variable and CLPT theory to model the buckling strength. Sebay et al. (2011) optimized the balanced and symmetric stacking sequences of rectangular composite plate subject to bi-axial loading in order to maximize the buckling strength. They increased the design space with the choice of dispersed layer. Erdal et al. (2005) stated that buckling is the critical failure mode in laminated composite plates subject to in-plane compressive loading and performed stacking sequence optimization to increase the buckling strength of the rectangular composite plate. The conventional layer angles were used as the design variables. Nicholas et al. (2012) stated that grouping of plies of same orientation will lead to interlaminar stresses even under in-plane loading and they also mentioned that grouping of plies can be minimized by using the reduced ply angle intervals.

The design variables like ply angle and ply thick have discrete values and thus the discrete optimization technique has to be applied to optimize the laminated composite plate. Various evolutionary algorithms are successfully applied to optimize the laminated composite structures in the last two decades. Rama Mohan Rao and Arvind (2005) used scatter search method which is one of the population based metaheuristic methods. In scatter search method, a reference set is generated from the population and then a subset is carefully chosen from this reference set. The procedure is improved by selecting the solutions and combining them. The result of the improvement can motivate the updating of the reference set and even updating of the population of solutions. Ozgur Erdal and Fazil (2005) and Mustafa and Fazil (2008) implemented an improved version of simulated annealing (SA) known as the direct search simulated annealing (DSA) to optimize the stacking sequence of composite structure. In simulated annealing, the optimization starts with a single solution and it leads to long time when the design space is huge but DSA works with a set of population like GA. The number of solutions in the populations is decided based on the dimension of the problem. The new populations are generated using the two mechanisms called as controlled generation and random generation. Aymeriah and Serra (2008) optimized the stacking sequence of laminated composite using ant colony optimization (ACO).

Suresh et al. (2007) carried out multi-objective optimization using the particle swarm optimization (PSO) to design the composite box-beam. GA is a population-based optimization technique [Phanden et al. 2012]. PSO is also population based optimization technique like GAs and stimulated by social behavior of bird flocking. Each solution in the search space is called as particle and it adjusts its position in the search space according to its own flying experience and the flying experience of other particles. Omkar et al. (2009) applied Quantum behaved Particle Swarm Optimization (QPSO) and Vector Evaluated Particle Swarm Optimization (VEPSO) respectively for the multi-objective optimization of composite structures. QPSO and VEPSO are co-variants of the Particle Swarm Optimization (PSO) algorithm. The main disadvantage in PSO is that the global convergence cannot be guaranteed and this problem is rectified in QPSO by the representing the particle's state with wave function instead of position and velocity. VEPSO method employs separate swarms for each of the objective and information migration among these swarms ensures an optimal solution with respect to all the objectives.

In this paper, a simple genetic algorithm and hybrid ant colony optimization proposed by Prabakaran et al. (2007) are used to optimize the rectangular composite plate. The effectiveness of these algorithms is compared in finding the optimum stacking sequence. The stacking sequence, in which the ply angle interval is reduced up to 5°, is used as the design variable. The in-plane strength computed using CLPT is chosen as the design constraint.

The paper is structured as follows. The optimization problem is formulated in section 2. The detailed Procedure of proposed optimization techniques are explained in section 3. The descriptions of results are given in section 4 and conclusions are presented in section 5.

2. PROBLEM FORMULATION

A rectangular composite plate subject to bi-axial compressive loading as shown in Fig. 1 is considered for this work. The simply supported boundary condition is applied on all four sides of the blade. The symmetric stacking sequences are assumed in order to remove the coupling effect. The ply thick and number of layers is kept constant and the stacking sequence is optimally varied to maximize the buckling strength. The in-plane strength of the plate computed using Tsai-Wu failure criteria is used as the design constraint. In order to reduce the grouping of plies, the maximum number of grouping of plies of same orientation is restricted to four.

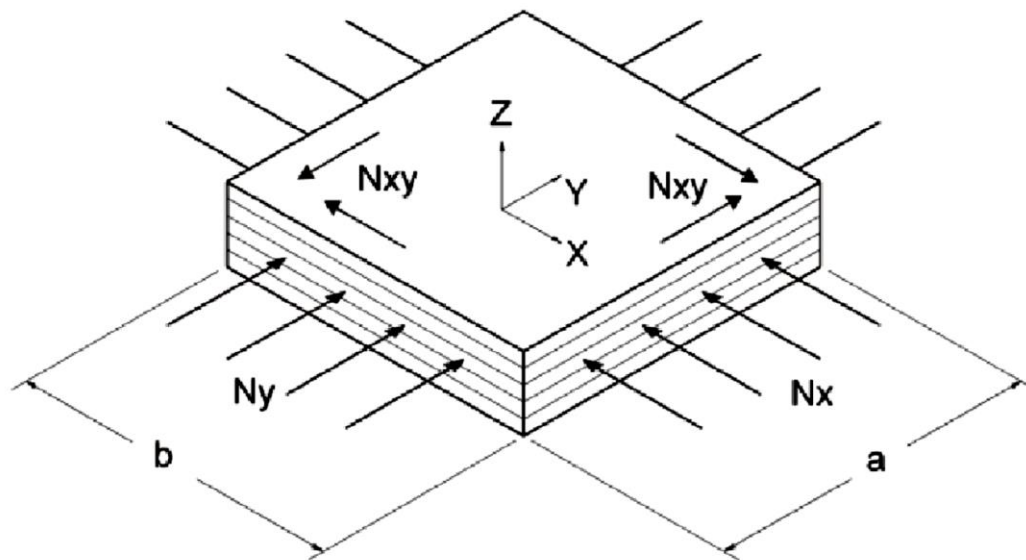


Figure 1: Geometry of the plate

2.1 The Classical Lamination Theory

The classical lamination theory is applied to evaluate the mechanical behaviour of the composite plate. The stress-strain relation of each ply is given in Eq.(1).

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_k = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{pmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{Bmatrix}_k \quad (1)$$

where \bar{Q}_{ij} are the off-axis stiffness components determined based on the ply orientations. The resultant forces and moments are obtained through the thickness integration of the stresses in each ply and it is given in Eq.(2).

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}_k = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} dz$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}_k = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} z dz \quad (2)$$

By substituting Eq.(1) in Eq. (2) we get,

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix}_k = \begin{pmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}_k + \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{Bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix}_k$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix}_k = \begin{pmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{pmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}_k + \begin{pmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{pmatrix} \begin{Bmatrix} k_{xx} \\ k_{yy} \\ k_{xy} \end{Bmatrix}_k \quad (3)$$

Where, A_{ij} , B_{ij} and D_{ij} are the components of extensional stiffness matrix, coupling stiffness matrix and bending stiffness matrix respectively. They can be computed using Eq.(4).

$$A_{ij} = \sum_{k=1}^m \bar{Q}_{ij} (h_k - h_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^m \bar{Q}_{ij} (h_k^2 - h_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^m \bar{Q}_{ij} (h_k^3 - h_{k-1}^3) \quad (4)$$

If the plate is loaded by the forces λN_x , λN_y , and λN_{xy} (λ is a scalar amplitude parameter), the buckling factor of the plate is computed using the Eq.(5).

$$\frac{\lambda_b}{\pi^2} = \frac{D_{11} \left(\frac{m}{a}\right)^4 + 2(D_{12} + D_{66}) \left(\frac{m}{a}\right)^2 \left(\frac{n}{b}\right)^2 + D_{22} \left(\frac{n}{b}\right)^4}{N_x \left(\frac{m}{a}\right)^2 + N_y \left(\frac{n}{b}\right)^2 + N_{xy} \left(\frac{mn}{ab}\right)} \quad (5)$$

2.2 Tsai-Wu Failure Criterion

As the Tsai-Wu criterion is a single criterion and has the relation between the strength components, it has been used by many researchers to compute the strength of the composite structure. The criterion is given in Eq. (6).

$$F_{LL}\sigma_{LL}^2 + 2F_{LT}\sigma_{LL}\sigma_{TT} + F_{TT}\sigma_{TT}^2 + F_{SS}\sigma_{LT}^2 + F_L\sigma_{LL} + F_T\sigma_{TT} > 1 \quad (6)$$

3. OPTIMIZATION PROCEDURE

3.1 Simple Genetic Algorithm

A simple genetic algorithm is applied to optimize the stacking sequence so that to maximize the buckling strength. It works with the operators like selection, crosses over and mutation to evolve a solution. Reproduction or selection operator is the first operator applied on population. The healthier chromosomes from the current population are selected to be parents to crossover and produce offspring. Out of many reproduction operators existing, roulette wheel selection, one of the most widely used selection operator is used in this work.

Crossover is a genetic operator which combines two parent chromosomes so as to produce the new chromosomes (offspring). Generally, the new chromosomes may be better than both of the parents for it takes the best characteristics from their parents. Crossover is taken place based on a user defined probability. Uniform crossover operator proposed by Nicholas et al. (2012) is used in this work.

After the crossover, mutation operator is carried out to maintain a genetic diversity from one generation of to the succeeding generation. Mutation also helps to prevent the population from stagnating at any local optima. Mutation is intended to prevent the search falling into a local optimum of the state space. The uniform mutation operator proposed by Nicholas et al. (2012) is used here. This swaps the chosen gene by a random value chosen between the specified ranges of that gene that gene. The procedure of simple genetic algorithm is shown in Fig.2.

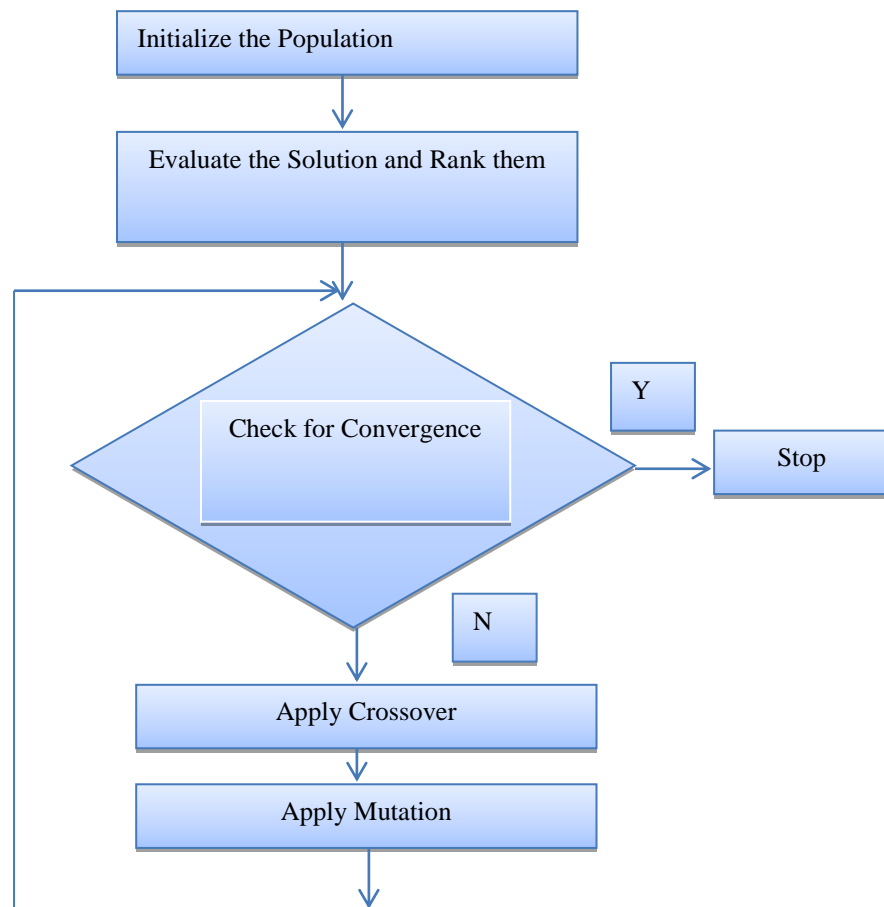


Figure 2: Flow chart of simple genetic algorithm

3.2 Ant Colony Optimization

Ant Colony Optimization procedure proposed by Prabhakaran et al. (2007) is applied here. Initial population is initialized by randomly generating 'N' solutions from the given design space. The fitness value of each solution is obtained and the superior and inferior solutions are identified from the current population. The inferior solutions are subject to global search in order to improve the fitness value. The global search comprises three

steps called as random walk, mutation and trail diffusion. The random walk is performed to replace the inferior designs using superior designs. The mutation is applied to further improve the replaced solutions by adding or subtracting with a probability relative to the mutation probability. The mutation step size is reduced or increased as per the following equation. Trail diffusion is used to improve the bottom most designs of the initial population.

Next the local search operation is performed so as to improve the superior designs. The pheromone value is set as unity for the initial population and similarly the age is chosen as 10. The new pheromone value and the age value are calculated based on the procedure given by Prabhakaran et al. (2007). The step by step procedure of ACO is given in Fig.3.

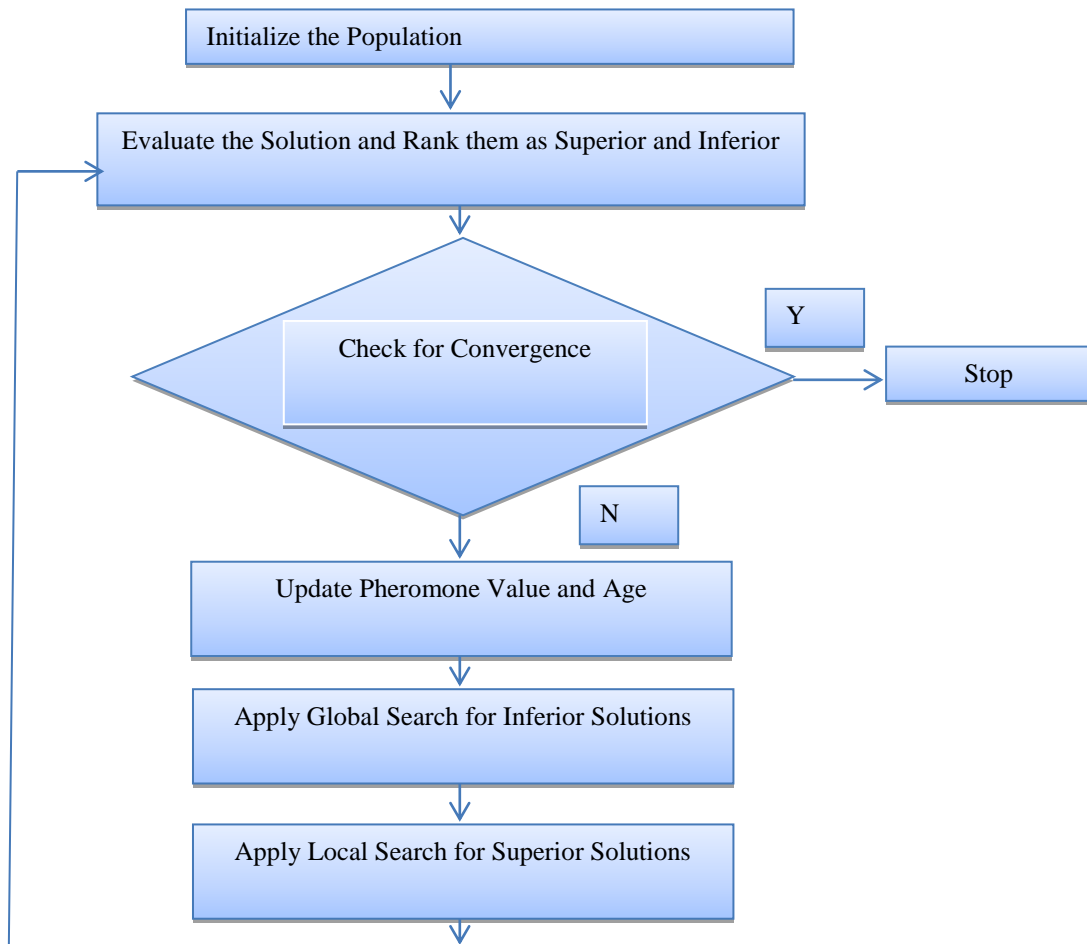


Figure 3: Flow chart of ant colony optimization

4. RESULTS AND DISCUSSION

The effectiveness of the proposed optimization techniques in the enlarged design space is compared here with a numerical example. A carbon/epoxy composite, with the material properties listed in Table 1, is applied for the rectangular composite plate.

Table 1: Material Properties

Material properties	Value
E_{11}	127.59 GPa
E_{22}	13.03 GPa
G_{12}	6.41 GPa
ν_{12}	0.3
S_{1T}	1500 MPa
S_{1C}	1200 MPa
S_{2T}	40 MPa
S_{2C}	246 MPa

The symmetric and balanced stacking sequences are chosen so as to remove the coupling stiffness components and to reduce 75% of the design variables. An equal bi-axial compressive load is applied on the structure as shown in Fig.1. The in-plane and buckling strength of the plate are computed using the Eqs. (5) and (6). The ply angle interval is reduced to 5° and the stacking sequences are optimized using the proposed GA and ACO. The results obtained using these optimization techniques are compared in Fig. (2).

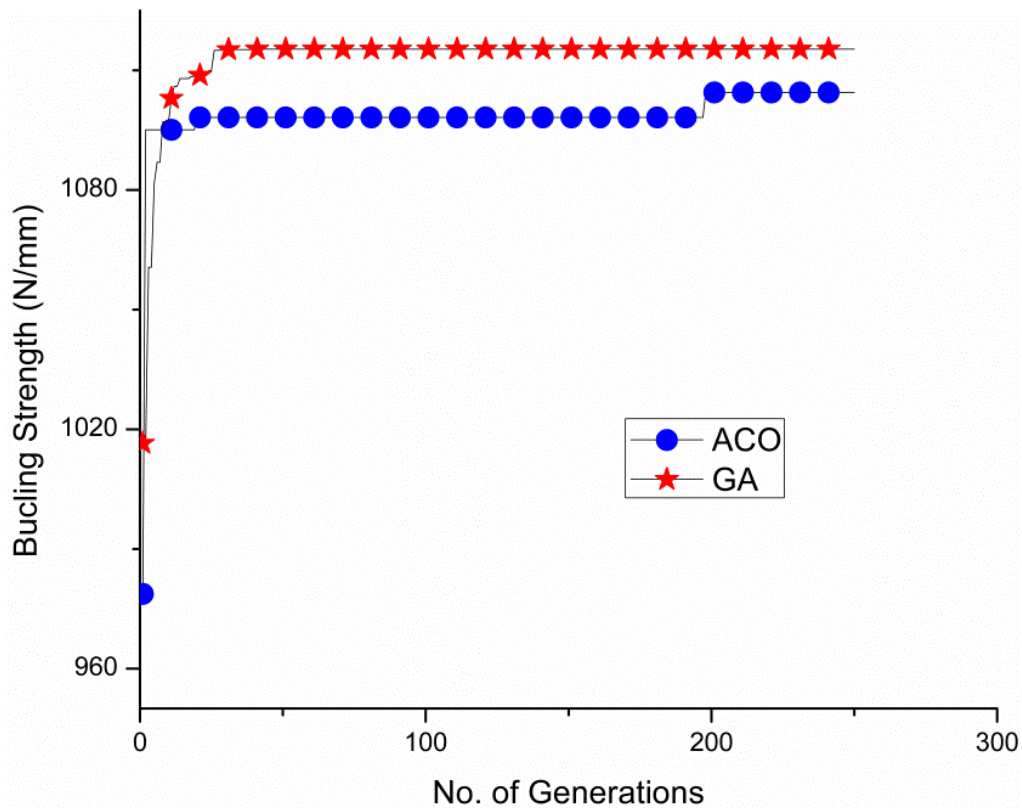


Figure 6: Comparison of GA and ACO

The results show that genetic algorithm gets converged faster than ACO and also find the better solution.

5. CONCLUSIONS

The buckling optimization of laminated composite was carried out by optimally varying the stacking sequences. The design space was increased more than nine times by reducing the ply angle interval to 5° . Hence, genetic algorithm and ant colony optimization, the two most success full optimization techniques were applied and compared. The numerical results show that genetic algorithm find the better solution than ACO. In addition, GA gets converged faster than ACO.

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