MATHEMATICAL MODULATION AND ERROR EVALUATION OF FREE FORM SURFACES

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Abstract: Now days, one of the critical factors in competitive technology is "time to market" along with a fool proof design. This critical factor indicates the entire product design cycle from concept to product design to prototype to manufacturing process design to actual implementation. To have command over this critical factor Computer aided designing (CAD) and manufacturing (CAM) is taking hold as a mean of speeding the time to market for new product development. The CAD approach is used in Rapid Prototyping techniques (RPT) for design and development of new products. Use of this CAD techniques in RPT techniques shorten the time to market and further for research and development of time of new products. In this seminar we are going to discuss about the properties and characteristics of freeform surfaces (B-Spline curve), the properties and characteristics of freeform surfaces (B-Spline curve), the Fused Deposition Modeling (FDM) the Coordinate Measuring Machine (CMM) and generate cloud point data, To determine the relation between the change in characteristics of the curve and the deviation values as the number of knot vectors (control points) are increased for designing component.

Keywords: **3D Printing, B-Spline curves, Computed Aided Design, Coordinate Measuring Machine, Error Evaluation, Free Form Surfaces, Rapid Prototyping**

1. INTRODUCTION

The ideas and the drawing of a design can sometimes be very far from the real world in which the product is used. By creating a rapid prototyping, it is possible to sit with a real version of the product and identify areas that are worth and which parts need to be revised, modified or scrapped. In the process, it may be possible to find glaring omissions that, on paper, or not visible. Furthermore, the creation of a prototype will allow the design team, not only to assist but also to test the product before entering the full production. By prototyping before production begins, it is possible to take an overview of the production process and see if steps can be modified, combined or eliminated. This is not only simplifies the production, but the cost of actual production to a minimum. Thereafter if there are problems in the production or perhaps process that can create problems for the final product, it's much better to see them before production begins. It can also help the design teams to determine the optimal method of production of injection moulding, silicon moulds, metal stamping, etc. without a prototype is just an idea. The rapid development of models will be successful only if the following apply, and only then:

a) Each prototype should provide answers to specific questions.
b) The prototype should be as detailed and complex as necessary to provide the required information.
c) If there are multiple ideas, then it is necessary that multiple prototypes be developed.
d) Decisions should be made during the development of the prototype and one should not await its final form.
e) The swifter the prototype development, the swifter the overall development of the new product.

The development of a new product is an iterative process that includes product prototyping for experimental evaluation and design modification. Reverse engineering is one of the solutions to convert the first conceptualization shape that will be successively coded in a mathematical expression with the help of physical model by rapid prototyping technique. The use of CMM is essential to manipulate the data for its successive valuation. Evident errors are common while comparing physical object with surface model.

The most common source of error in RP processes are categorized into mathematical error, process related error and material related error. The errors involved during the data preparation stage are identified as tessellation, slicing orientation and location. Facet approximation and staircase effect also belong to data preparation errors. Principle process related error is shrinkage and distortion using the material. It is very important to understand the error distribution and error transfer mechanism in order to asses, predict and control the final geometric accuracy of the part. Error interaction between the data preparation and fabrication process are equally important.

In this work a new methodology is used to reduce the data preparation error by increasing number of critical points on the 3-D CAD model of the free-form surface of the curve in such a way that the variation of slope and curvature of the curve is reduced at critical points. Due to this discontinuity of the curve at critical points is reduced, resulting in reduced deviation of the curve.

2. METHODOLOGY

a. Design of freeform surfaces in CATIA V5 using B-Spline curve equation
b. Producing 3D model using rapid prototyping process using the fused deposition modelling (FDM) technique.
c. Measuring the produced components using Co-

ordinate measuring machine (CMM) and generating cloud point data.
d. Converting the cloud point data to CAD file.
e. Overlapping the CMM CAD Model with original CAD model and finding the deviation values.
f. Deviation values for all the initial and offset components are measured and compared.
g. Relation between the change in characteristics of the curve and the deviation values for the initial components are determined.

3 RAPID PROTOTYPING

3.1 Principle

Rapid prototyping is essentially a part of automated fabrication, a technology that lets us make three-dimensional parts from digital designs. There are several advantages of automated fabrication over manual fabrication and moulding processes. Some of these advantages are computer-aided design, quick design changes, and precise dimensioning. The rapid prototyping process (layered manufacturing) falls into the additive fabrication category.
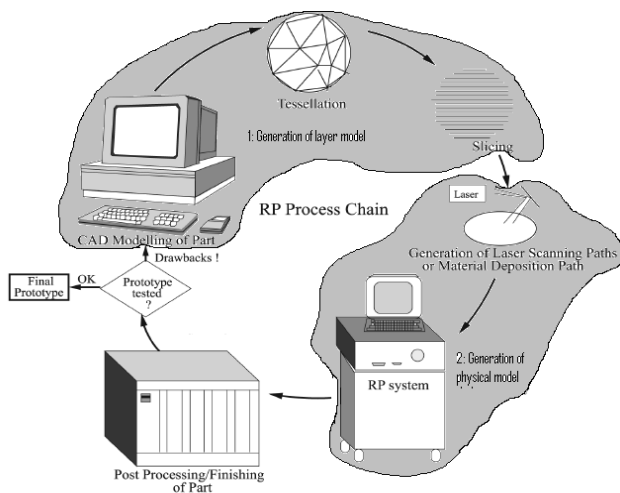

Fig. 1. Showing RP process and fundamental steps. Examples of RP processes include SLA, FDM, LOM, and SLS, solid ground curing (SGC), direct shell production casting (DSPC) and 3D printing (3DP).

3.1.1 Stereo Lithography Process

A low-power ultraviolet laser beam is moved at the surface of a vat of photo curable liquid polymer, producing a single layer of solidified resin as shown. Each time the platform is lowered a distance equal to layer thickness and resin above platform cured. Separate support structure has to be added for overhanging structures.

3.1.2 Fused Deposition Modelling

Fused deposition modelling (FDM) is an extrusion-based rapid prototyping (RP) process, although it works on the same layer-by-layer principle as other RP systems is shown in Figure 3. Fused Deposition Modelling relies on the standard .STL data file for input, and is capable of using multiple build materials in a build/support relationship.

The FDMs can be equipped to build with investment casting wax, acrylonitrile butadiene styrene (ABS) plastic, medical grade ABS thermoplastic, and/or Elastomer, although the ABS is currently used the most.

3.2 Rapid Prototyping Methodology

All prototypes made with both the current and evolving RP processes have several features in common. A solid or surface CAD model is electronically sectioned into layers of predetermined thickness. These sections define the shape of the part collectively. Information about each section is then electronically transmitted to the RP machine layer by layer. The RP machine processes materials only at "sold" areas of the section. Subsequent layers are sequentially processed until the part is complete. It is this sequential, layered, or lithographic approach to parts manufacturing that defines RP. The RP process basically uses the following steps to make prototypes.
a. Create a CAD model of the design
b. Convert the CAD model to .STL file format.
c. Slice the .STL file into 2D cross-sectional layers.
d. Grow the prototype
e. Post processing

3.2.1 Creation of Solid Models

The first step in creating a prototype is the creation of a CAD solid model. A solid is a volume completely bounded by surfaces, which means the edges of all surfaces must be coincident with one, and only one, other surface edge. Unlike wire-frame and surface modelling, solid modelling stores volume information. A CAD solid model not only captures the complete geometry of an object, it can also differentiate the inside and the outside of the space of that object. Solid models can be created using a CAD software package such as Auto CAD, Pro/Engineer, CATIA, Solid Works, or many other commercially available solid modeling programs.

3.2.2 Creation Of .STL File

Once a solid model is created and saved, it is then converted to a special file format known as .STL (stereo lithography). This file format originated from 3D systems. Though not ideal, it is sufficient to meet the needs of today's rapid prototyping technology, which generally build mono material parts. The success of this file format has been impressive. Today, a decade later, the .STL file format remains the de facto standard for the rapid prototyping industry. The success of the .STL file format

is due to its sufficiency, its simplicity, and its monopoly. Its mathematical sufficiency stems from the fact that it describes a solid object using a boundary representation technique. A .STL file format represents the virtual CAD model of the object to be prototyped as a collection of triangular facets. These triangular facets, when taken together, describe a polyhedral approximation of the objects' surface, that is, a polyhedral approximation of the boundary between material and nonmaterial. In short, a .STL file is nothing more than a list of *x, y,* and *z*. Co-ordinates triplets are that describe a connected set of triangular facets. It also includes the direction of the normal vector for each triangle, which points to the outer surface of the model. The output of .STL file formats can be expressed in binary or ASCII format.

3.2.3   Slicing Of File

Once the .STL files are verified to be error-free, the RP system computer analyses the .STL files that define the model to be fabricated and slices the model into cross-sections. The cross-sections are systematically recreated through the solidification of either liquids or powders or fusing of solids, to form a 3D model. The model is sliced into the thinnest layer as they have to be very accurate.

3.2.4   Making or Growing the Prototype

For most RP system, this step is fully automated. The part is submitted to the RP computer and the machine run until the part is complete. RP machines build one layer at a time from polymers, paper, or powdered metals. Most machines are fairly autonomous needing little human intention. Build times vary depending on size and number of part required.

3.2.5   Post Processing

The final step in rapid prototyping is post processing. It essentially consists of part removal and cleaning and of post curing and finishing. This step generally involves manual operations where an operator does the post processing with extreme care. Otherwise, the part may be damaged and may need to be prototyped again. The tasks for post processing are different for different prototyping systems. Part removal and cleaning refers to taking the prototype out of the prototyping machine and removing excess materials, including support materials, which may have remained on the part. With SLA parts, excess resins residing in entrapped areas need to be removed. Likewise, excess powder for SLS parts and excess wood like blocs of paper for LOM parts need to be removed. Post curing is a task that is usually only needed for SLA and SLS parts. Post curing is needed to complete the solidification process and to improve the mechanical properties for the prototype. For simple part application, finishing involves basic cleaning such as sanding or machining to remove additional materials.

3.3   Error Induced In Rapid Prototyping

Although the original RPTs were developed to provide style models, current RPTs are increasingly regarded as manufacturing technologies for tooling and production parts. All manufacturing technologies for tooling and production parts must guarantee the accuracy of geometric dimensions and repeatability. The subject of geometric accuracy of parts is thus under intense debate as practitioners argue where and under what circumstances a rapid prototyping machine provides enough accuracy to be suitable for making tools. Yan and GU categorized the most common sources of errors in rapid prototyping processes into mathematical error, process-related error and material-related error. Chalasani and Bagchi identified tessellation, slicing orientation and slicing location as the three primary sources of error in the data preparation stage. Also, errors in the data preparation process were found in facet approximation, staircase effect and containment problem, and in laser beam path planning algorithms. Process-related errors affect the shape of the layer in the x, y-plane and along the z-axis as well as the overall 3D shape. The physical and chemical characteristics of the materials in the process of solidification involving liquid and powder materials have been studied by many workers who showed that the principal process-related errors are shrinkage and distortion in sing these materials. For solid sheet materials, little is reported for analyzing component geometrical error relating to the fabrication process. From the viewpoint of component production process in layered manufacturing, it is very important to understand the error distribution and error transfer mechanisms in order to assess predict and control the final geometric accuracy of the part. A geometrical model is used for assessing the effects of various errors and their accumulation and the final part geometry in RPT considering error interactions between the data preparation process and fabrication process.

3.3.1   Data Preparation Errors (DPE)

In the RPT process, each 2 ½ -D layer is generated as "sweep" of a planar profile by depositing material on the interior of a 2D slice. Since the whole part is manufactured in this manner, the boundary of the part created is a stepped approximation of the boundary of the ideal part. As a result of this, compared with the parts created in the conventional manufacturing technologies such as milling and turning, parts produced in layered manufacturing exhibit a staircase effect. The error resulting from the staircase effect can be affected by the containment mode which describes the geometrical arrangement between the ideal part geometry and the sliced part contours. There are three containment modes to approximate the ideal part in the RPT process. These are referred to as the positive situation (the sliced part profile slightly exceeding the ideal profile) and the hybrid situation. The approximating accuracy in the slicing procedure also depends on the slicing

orientation, location and slicing thickness as reported in. These errors are not related to the actual fabricating process and only depend on the fabricating method selected and slicing parameters used. These errors are therefore lumped as data preparation error (DPE) and they can be predicted theoretically by analyzing the extent of the staircase effect for a given containment situation and slicing parameters selected.

3.3.2   Fabricating Process Errors

In the actual RPT part-building process there are additional errors which are affected by many process-related parameters, e.g. the machine path control accuracy, tool scan speed uniformity, tool shape stability, platform control accuracy, material properties, part thermal distortion, material feed uniformity, and part thermal shrinkage and distortion. These errors interact with the DPE and when combined they determine the final geometrical accuracy of the part produced. There are three possible effects of these fabrication errors on the final geometry of the part:

    a. Geometrical errors generated in the x- and y-directions only which are primarily associated with the scan path and tool shape control accuracy as shown.

    b. Geometrical errors in the z-direction which are mainly related to the platform displacement accuracy and materials thickness uniformity (e.g. in LOM and SLS processes) as shown.

    c. Geometrical errors in all directions as a result of scan speed variation (acceleration and deceleration), material flatness variation (e.g. in LOM process), thermal distortion of the part and other assembly errors. It can be seen that the errors resulting from the actual part building process can alter the extent of theoretical errors relating to the staircase and containment problem defined by the fabrication method. For the purpose of error analysis, all the errors generated during the part-building process are now defined as the disturbance error.

3.4 Freeform Surface

The technology has encompassed two main fields either creating aesthetic (Class A surfaces) or by CAD software. By creating aesthetic (Class A surfaces) applications are such as car bodies and consumer product outer forms, or technical surfaces for components such as gas turbine blades and other fluid dynamic engineering components.

CAD software packages use two basic methods for the creation of surfaces. The first begins with construction curves (splines) from which the 3D surface is then swept (section along guide rail) or meshed (lofted) through.

The second method is direct creation of the surface with manipulation of the surface poles/control points. Freeform

surface or freeform surfacing is used in CAD and other computer graphics software to describe the skin of a 3D geometric element. Freeform surfaces do not have rigid radial dimensions, unlike regular surfaces such as planes, cylinders and conic surfaces. They are used to describe forms such as turbine blades, car bodies and boat hulls. Initially developed for the automotive and aerospace industries, freeform surfacing is now widely used in all engineering design disciplines from consumer goods products to ships. Most systems today use non-uniform rational B-spline (NURBS) mathematics to describe the surface forms

Implicit functions and parametric functions are the two most common forms of representing curves and surfaces in geometric modelling.

An implicit function has the form:

$F(x, y, z) = 0$

  A parametric function has the form:

$$P(u) = [x(u), y(u), z(u)](u_{min} < u < u_{max}]$$

Parametric functions hold a number of advantages over implicit functions. Some of the most important ones include:

    a. Points can be evaluated reasonably fast by numerically stable and accurate algorithms.

    b. There are more degrees of freedom than implicit functions have.

Represented with parametric functions, B´ezier, B-spline and NURBS are three widely used

Mathematical models for freeform curves and surfaces in current CAD applications. B´ezier, B-spline and NURBS all belong to an important geometric class called splines. This section gives a glance of the three mathematical models.

B spline is the generalisation of B'ezier curve. B spline curve with degree p is defined by n+1 control points P_0..., P_n and a knot vector of m+1 knots:

$$U = \{u_0, u_1, ... u_m\}$$

Where U is a non-decreasing sequence with u_i 2 [0, 1], and n, m and p must satisfy:

$p = m - n - 1;$
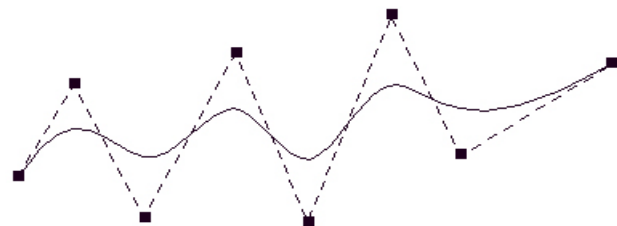

Fig. 2. B-spline curve with 8 control points

The B-spline parametric curve function is of the form:

$$C(u) = \sum_{i=0}^{n} P_i N_{i,p}(u)$$

$$N_{i,p}(u)$$

are the basis function of the B-splines, defined by:

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \leq u \leq u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

A B-Spline surface is an expansion of B-spline curves in two directions, with corresponding control points, knot vectors, and univariate B-spline functions. The surface is defined by where the k1 are the orders (degree+1) of the B-spline surface in both directions. The Ni, k(u) are the polynomial B-spline basis functions of degree k-1 in the u parameter direction, and Nj, l(v) are the basis functions of degree l-1 in the v direction. If the knot vector of a B-spline curve is just (p+1) zeros followed by (p+1) ones, this B-spline curve reduces to a Bezier curve. Therefore actually B-spline curves are a generalization of B´ezier curves, and the same applies for surfaces.

Here are some important properties of B-spline curves/surfaces:

a. Smoothness: A p-degree B-spline curve/surface has continuity of (p-k) at the knots of multiplicity k (k knots clamped together). Figure 2.5 shows a 4-degree B-spline curve. It is easy to calculate that the curve at the three multiple knots has continuity of 2, 1, 0 respectively. For continuity 1, the corresponding point lies on the control polygon, while the curve pass through a control point for continuity 0, which results in a visual discontinuity.

b. Strong convex hull: The curves/surfaces are contained in the convex hulls of their defining control points. Especially, for a B-spline curve, if u 2 [ui0, uio+1), p $\leq$ i0 $\leq$ m−p−1, then the curve C(u) is in the convex hull of the control points Pi, io-p $\leq$ i $\leq$ i0. For a B-spline surface with a knot vector {u0, u1,..., um} in u direction and a knot vector{v0, v1,..., vn} in v direction, if (u, v) 2 [ui0 , ui0+1)×[vj0 , vj0+1), then the surface S(u, v) is in the convex hull of the control points

c. Affine invariance: An affine transformation is any transformation that preserves collinearity (i.e., all points lying on a line initially still lie on a line after transformation) and ratios of distances (e.g., the midpoint of a line segment remains the midpoint after transformation). In general, an affine transformation is a composition of rotations, translations, dilations, and shears, and they are all subclasses of projective transformations. For a B-spline curve/surface, one can apply the affine transformations by just applying them to the control polygon/net.

d. Local modification scheme: Moving Pi changes a B-spline curve only in the interval [ui, ui+p+1], because Ni,p(u) = 0 for u 62 [ui, ui+p+1]; similarly, moving Pi,j only affects a B-spline surface in rectangle [ui, ui+p+1)×[vj , vj+q+1), because Ni, p(u) Nj, q(v) is zero if (u ,v) is outside of the this rectangle.

4. COORDINATE MEASURING MACHINES

4.1 About CMM

Coordinate Measuring Machine (CMM) is a device for measuring the physical geometrical characteristics of an object. This machine may be manually controlled by an operator or it may be computer controlled. Measurements are defined by a probe attached to the third moving axis of this machine. The typical "bridge" CMM is composed of three axes, an X, Y and Z. These axes are orthogonal to each other in a typical three dimensional coordinate system. Each axis has a scale system that indicates the location of that axis. The machine will read the input from the touch probe, as directed by the operator or programmer. The machine then uses the X, Y and Z coordinates of each of these points to determine size and position. Typical precision of a coordinate measuring machine is measured in Microns. The figure 5 show the Coordinate Measuring Machine (CMM)

A coordinate measuring machine (CMM) is also a device used in manufacturing and assembly processes to test a part or assembly against the design intent. By precisely recording the X, Y, and Z coordinates of the target, points are generated which can then be analyzed via regression algorithms for the construction of features.

$$p_i, j, i_0 - p \leq i \leq i_0 \& j_0 - q \leq j \leq j_0$$

Fig. 3. Coordinate Measuring Machine (CMM)

4.2 TECHNIQUES FOR REDUCING THE ERROR

4.2.1 Increasing The Number Of Control Points

The general 3-D B-Spline curve Equations used for generating the curve:

$$S(X) = C_{11}(X - X_1)^3 + C_{12}(X - X_1)^2 + C_{13}(X - X_1) + C_{14}$$

For generating the curve using 6 points, general equation is substituted with the boundary conditions and limits, coefficients C11, C12, C13 and C14 are obtained.

C_{11}	C_{12}	C_{13}	C_{14}
-0.0002	0.0029	0.5900	0
-0.0002	0.0081	0.5049	9.5000
-0.0009	-0.0180	0.1164	14.5000
0.0006	-0.0336	-0.1705	14.4300
0.0006	-0.0092	-0.7481	7.5000

Table. 1. Coefficients C11, C12, C13, C14

Using these coefficients, the B-spline curve equations generated are:

$$S(x) = -0.0002(x^3) + 0.0029(x^2) + 0.5900x + 0$$
$$S(x) = -0.0002(x - 16.45)^3 - 0.0081(x - 16.45)^2$$
$$+ 0.5049(x - 16.45) + 9.5000$$
$$S(x) = -0.0009(x - 31.36)^3 - 0.0180(x - 31.36)^2$$
$$+ 0.1164(x - 31.36) + 14.500$$

$$S(x) = 0.0006(x - 36.92)^3 - 0.0336(x - 36.92)^2$$
$$- 0.1705(x - 36.92) + 14.43$$

$$S(x) = 0.0006(x - 50.4)^3 - 0.0092(x - 50.4)^2$$
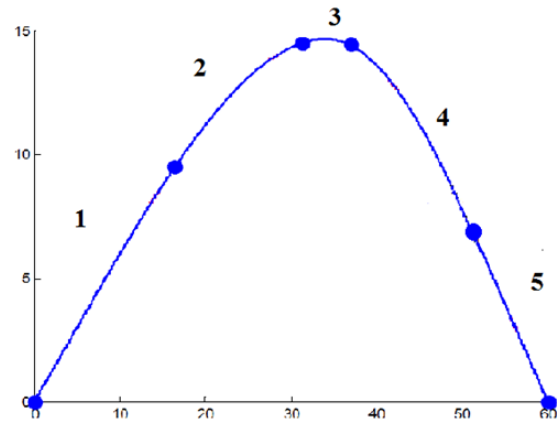$$- 0.7481(x - 50.4) + 7.500$$

Fig. 4. B-spline curve for 6 control points

Similarly the curves with 11 points, 16 points and 19 points are generated using the general 3-D B-Spline curve equation with suitable boundary conditions and limits. Consequently curves are generated using 10 equations, 15 equations and 18 equations respectively.

4.3 Offset Method

During the conversion of CAD model into STL file there are number of tessellation errors and data transformation errors. To minimize these errors, for the given curves an offset values are added to common vertices in STL file. The offset values have to be given by the user depending on the application. Offset values are given as 0.01mm, 0.02mm and 0.05mm. In this case the standard deviation values of the curve will be considered as offset values.

5. RESULTS AND DISCUSSION

5.1 Deviation values obtained by increasing number of control points on the curve

After producing the CAD model from point cloud data, this CAD model and original CAD models are overlapped to know the deviation of the curve at different points. For different curves which are produced by introducing different control points, deviation values are tabulated, which are shown .

X	Y	6 points		11 points	
		Error on right side curve	Error on left side curve	Error on right side curve	Error on left side curve
0	0	0.1511	0.1573	0.1607	0.0865
4.1125	2.4615	0.0118	0.0102	0.0173	0.0175
8.225	4.9376	0.0092	0.0103	0.0197	0.0116
12.3375	7.34495	0.0049	0.0152	0.0547	0.0117
16.45	9.500	0.0159	0.0015	0.0529	0.0069
20.1775	11.5763	0.0270	0.0078	0.0368	0.0342
23.905	12.73099	0.0090	0.0217	0.0406	0.0513
27.6325	13.85348	0.0026	0.0290	0.0360	0.0612
31.36	14.5	0.0820	0.0162	0.0108	0.0179
32.75	14.624	0.0613	0.0163	0.0058	0.0278
34.14	14.664	0.0745	0.0161	0.0062	0.0394
35.53	14.607	0.0638	0.0386	0.0280	0.0634
36.92	14.43	0.0940	0.0250	0.0464	0.0653
40.29	13.496	0.0987	0.0545	0.0531	0.0551
43.66	11.932	0.0795	0.0264	0.0141	0.1196
47.03	9.8919	0.0777	0.0519	0.0460	0.0951
50.4	7.5	0.1143	0.0026	0.0443	0.0810
52.8	5.6598	0.1641	0.0264	0.0751	0.0374
55.2	3.7635	0.1648	0.0712	0.0840	0.0454
57.6	2.271	0.1466	0.1056	0.0683	0.0005
60	0	0.3073	0.3110	0.1955	0.2344
AVERAGE		0.083814286	0.048323881	0.05220476	0.05539046
STANDARD DEVIATION		0.053806	0.069154	0.04638	0.050475

Table. 2. Deviation Values

X	Y	16 points		19 points	
		Error on right side curve	Error on left side curve	Error on right side curve	Error on left side curve
0	0	0.1629	0.1297	0.1927	0.0787
4.1125	2.4615	0.0315	0.0608	0.0214	0.0315
8.225	4.9376	0.0163	0.0501	0.0148	0.0224
12.3375	7.34495	0.0007	0.0204	0.0076	0.0202
16.45	9.500	0.0362	0.0045	0.0100	0.0129
20.1775	11.5763	0.0006	0.0139	0.0147	0.0141
23.905	12.73099	0.0366	0.0183	0.0232	0.0249
27.6325	13.85348	0.0008	0.0484	0.0190	0.0129
31.36	14.5	0.0166	0.0519	0.0356	0.0112
32.75	14.624	0.0538	0.0364	0.0418	0.0137
34.14	14.664	0.0493	0.0617	0.0371	0.0015
35.53	14.607	0.0559	0.0517	0.0608	0.0211
36.92	14.43	0.0496	0.0598	0.0518	0.0240
40.29	13.496	0.0543	0.0661	0.0473	0.0232
43.66	11.932	0.0180	0.1000	0.0103	0.0368
47.03	9.8919	0.0279	0.0763	0.0012	0.0472
50.4	7.5	0.0583	0.0799	0.0490	0.0574
52.8	5.6598	0.0577	0.0703	0.0315	0.0323
55.2	3.7635	0.0506	0.0548	0.0725	0.0116
57.6	2.271	0.0883	0.0436	0.0490	0.0145
60	0	0.2269	0.1778	0.2091	0.2171
AVERAGE		0.05203809	0.060780952	0.047638095	0.03472381
STANDARD DEVIATION		0.052274	0.038336376	0.053146361	0.044293

Table. 3. Deviation Values

Points	Slope	Curvature
0 4.1125	0.568372 0.614483	0.0215763 0.0008493
4.1125 8.225	0.614513 0.57544	0.000864 -0.019863
8.225 16.45	0.5754 0.595201	-0.01984 0.02457
16.45 20.1775	0.594786 0.436162	0.02454 -0.10965
20.1775 23.905	0.435747 0.262635	-0.10988 0.016996
23.905 31.36	0.262621 0.122496	0.016988 -0.05458
31.36 32.75	0.123312 0.058781	-0.05436 -0.03848
32.75 34.14	0.05878 -0.00450	-0.03848 -0.052574
34.14 36.92	-0.00449 -0.17049	-0.05254 -0.06688
36.92 40.29	-0.17051 -0.37809	-0.0669 -0.05608
40.29 43.66	-0.37771 -0.54240	-0.05608 -0.041663
43.66 50.42	-0.54238 -0.74553	-0.04166 -0.001921
50.42 52.81	-0.74756 -0.78204	-0.04166 -0.001921
52.81 55.23	-0.78206 -0.79981	-0.00952 -0.00112
55.23 60	-0.79483 -0.75983	-0.00112 0.015699

Table. 4. Deviation Values

Points	Slope	Curvature
0 8.225	0.596105 0.590947	0.04308 -0.00556
8.225 16.45	0.591017 0.094701	-0.0554 -0.0652
16.45 23.905	0.504871 0.348381	-0.0154 -0.0265
23.905 31.36	0.347862 0.115318	-0.02672 -0.03566
31.36 34.14	0.115752 -0.005075	-0.03554 -0.0573
34.14 36.92	-0.00511 -0.170723	-0.0574 -0.0677
36.92 43.66	-0.17084 -0.5415	-0.06782 -0.04218
43.66 50.4	-0.5416 -0.7472	-0.04218 -0.01884
50.4 55.2	-0.74733 -0.79541	-0.01886 -0.00117
55.2 60	-0.79541 -0.7586	-0.00118 0.016503

Table. 5. Deviation values

X	Y	Offset 0.01		Offset 0.02		Offset O.05	
		Error on right side curve	Error on left side curve	Error on right side curve	Error on left side curve	Error on right side curve	Error on left side curve
0	0	0.1767	0.1009	0.0748	0.0024	0.1945	0.1746
4.1125	2.4615	0.0268	0.0358	0.0774	0.0680	0.0536	0.0632
8.225	4.9376	0.0391	0.0075	0.0973	0.0542	0.0134	0.0063
12.3375	7.34495	0.0901	0.0115	0.1562	0.0656	0.0493	0.0012
16.45	9.500	0.0855	0.0302	0.1815	0.1049	0.0541	0.0390
20.1775	11.5763	0.0920	0.0559	0.2048	0.1335	0.0582	0.0378
23.905	12.73099	0.0557	0.0611	0.2428	0.1209	0.0597	0.0700
27.6325	13.85348	0.0579	0.1044	0.1825	0.1507	0.0667	0.1008
31.36	14.5	0.0213	0.0590	0.2099	0.1020	0.0481	0.1022
32.75	14.624	0.0473	0.0166	0.2270	0.1159	0.0156	0.0691
34.14	14.664	0.0579	0.0502	0.2242	0.1216	0.0612	0.1034
35.53	14.607	0.0486	0.0565	0.1992	0.1355	0.0109	0.0803
36.92	14.43	0.0354	0.0717	0.1987	0.1289	0.0696	0.1192
40.29	13.496	0.0456	0.0552	0.1281	0.0244	0.0604	0.1123
43.66	11.932	0.0614	0.0286	0.0605	0.0879	0.0336	0.1091
47.03	9.8919	0.0121	0.0917	0.0844	0.0000	0.0752	0.1412
50.4	7.5	0.0175	0.0460	0.1500	0.0891	0.0525	0.1239
52.8	5.6598	0.0525	0.0176	0.2322	0.1235	0.0045	0.0935

55.2	3.7635	0.1252	0.0083	0.2849	0.1243	0.0193	0.0624
57.6	**2.271**	**0.0740**	**0.0154**	**0.2857**	**0.1113**	**0.0701**	0.0910
60	0	0.0849	0.1802	0.0791	0.1645	0.1185	0.1681
Average		0.06226190	0.052585714	0.1705 33333	0.0966 23	0.005 661904	0.08 898095
Standard deviation		0.0374 70909	0.0404 365	0.0688 52944	0.0448 3843	0.040 310127	0.04 461828

Table. 6. Deviation Values

Fig. 7 Mean deviation values for the components with increasing number of control point on the curve

Fig. 5. Deviation values at different points for increasing no. of control points

The deviation values for the three components are compared and tabulated. Using these values a graph is made for the offset components.

Fig.8. Standard deviation values for the components with increasing number of control point on the curve

The mean and standard deviation values all the three offset components with increasing offset value are shown on the graphs.

Fig. 6 Deviation values at different points for increasing offset value

The mean and standard deviation values all the initial four components and the three offset components are observed and graph are drawn to see the variation and change in the values.
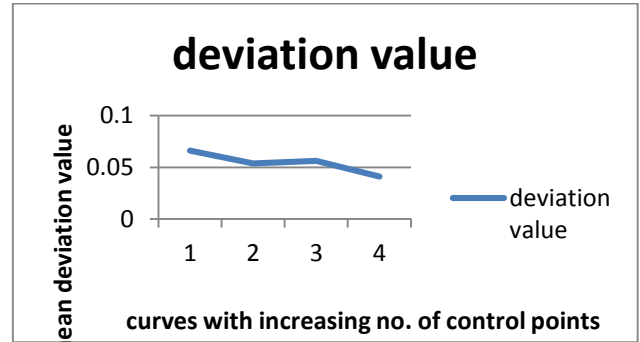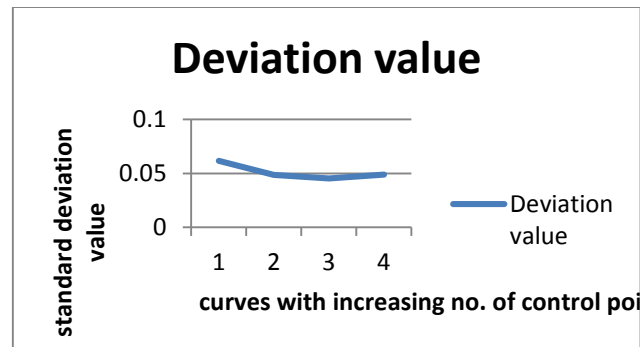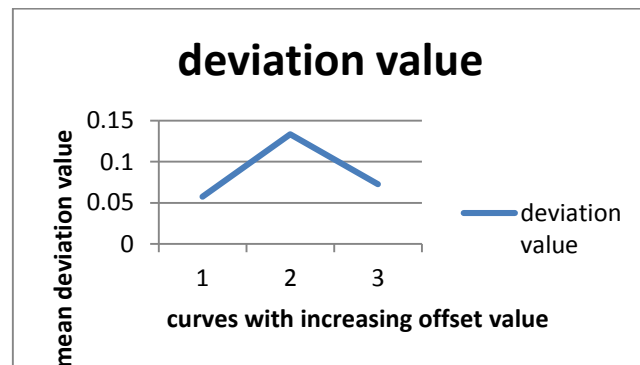

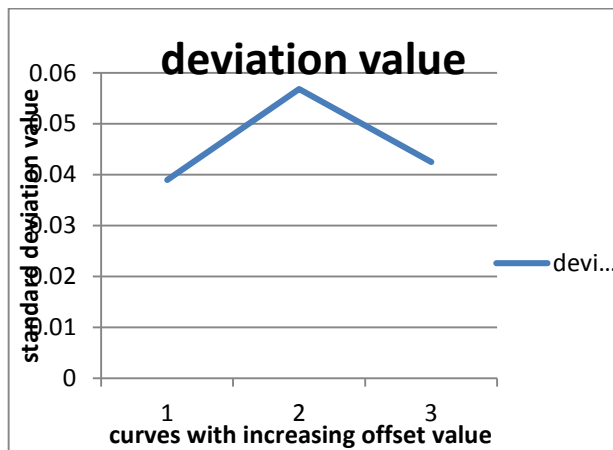Fig. 9 Mean deviation values for the three offset components with increasing offset value

Fig. 10.Standard deviation values for the three offset components with increasing offset value

6. CONCLUSION

a. For 3-D free-form complex curved surface which is represented by bi-cubic parametric b-spline curve, the number of control points on the curve is increased in such a way that the variation of slope and curvature of the curve at critical points is reduced, resulting in reduced discontinuity of the curve. There by improved smoothness of the curve with the reduced deviation.

b. For the common vertices of the STL file of the model, different offset values will be added resulting in reduced deviation of the free-form surface of the objects and increased smoothness of the curve.

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neighbor. The results show that the use of evidence theory during the quantization step and Huffman coding is an improvement of the quality of the reconstructed images.

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