

ANALYSIS OF PHOTONIC BAND STRUCTURE IN ONE DIMENSIONAL PHOTONIC CRYSTALS CONTAINING SINGLE NEGATIVE MATERIALS USING PWE AND FDTD METHOD

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Abstract—A photonic crystal (PC) is said to be an artificially periodic layered structure that is known to possess photonic band gaps (PBGs) at certain frequency ranges. In this paper we have analysed one dimensional band structure by computational method in single negative material photonic crystals using plane wave expansion and finite difference time domain methods.

Keywords—Forbidden gap, PWE, FDTD, Eigen states, Photonic band gap, single-negative (SNG) materials

I. INTRODUCTION

Some of the similarities between Photonic crystals [1] and solid-state physics include similar lattice structure, behaviour of photons is similar to electron and hole behaviour, due to the lattice periodicity both provide the band gap and most important is the determination of Eigen functions in a Photonic crystal is very similar to calculation of the quantum mechanical particle wave functions in the solid-state. Vectorial property of EM fields is the crucial reason to justify why photonic bands precisely predict the behaviour of light. The photonic crystals [2] are known to be the special class of optical media with periodic modulation of the refractive index and permittivity.

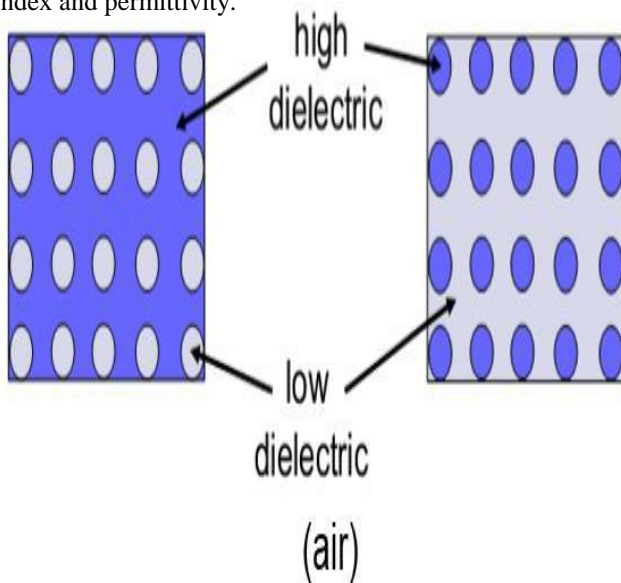


Fig. 1 The key point of photonic crystal is structures with periodic variations in the dielectric constant.

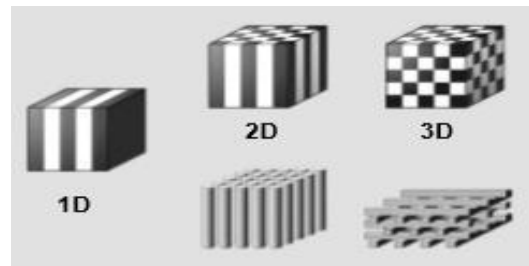


Fig. 2 Schematic view of photonic crystals with different dimensionalities

The paper is divided into: Section II includes the design of basic equations, Section III shows the various techniques involved in realization of the results, Section IV contains the heart of entire writing and discussions i.e. numerical results and discussions and Section V consists of summary and future research followed by section VI that contains the conclusion of the paper.

II. BASIC EQUATIONS

These are the differential form of Ampere's Law and Faraday equations, respectively. These two equations are correspondingly used to solve numerical calculation in PWE and FDTD method.

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t} + \sigma E$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

Where, E and H are the electric field and magnetic field, respectively; ϵ is the electrical permittivity and μ is the magnetic permeability; σ is the electric conductivity loss; t is the time.

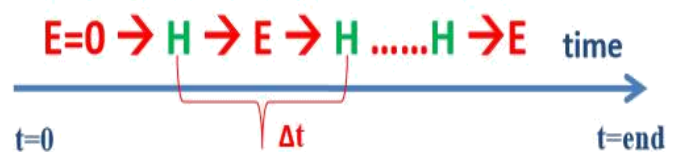


Fig.3 General FDTD field update procedure

III. TECHNIQUES INVOLVED

The paper was characterized by using two techniques mainly as

- Plane Wave Expansion (PWE) method
- Finite Difference Time Domain (FDTD) method

Conventional all-dielectric photonic crystals are made of the so-called double-positive (DPS) materials with both positive permittivity and permeability but our whole paper is characterized for the single-negative (SNG) [3] materials. There are two kinds of different SNG materials [4]. One is called the epsilon-negative (ENG) material, which has a negative permittivity and a positive permeability. The other called mu-negative (MNG) with a negative-permeability and a positive permittivity.

The material equations for non-magnetic medium take the following form:

$$\mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{H}(\mathbf{r}, t),$$

$$\mathbf{D}(\mathbf{r}, t) = \epsilon_r(\mathbf{r}) \epsilon_0 \mathbf{E}(\mathbf{r}, t)$$

To start the computation of 1D structure characteristics, we derived the wave equation from the system of Maxwell's equation.

$$\nabla \times \mathbf{E}(\mathbf{r}, t) = -\frac{\partial \mu_0 \mathbf{H}(\mathbf{r}, t)}{\partial t},$$

$$\nabla \times \mathbf{H}(\mathbf{r}, t) = \frac{\partial \epsilon_r(\mathbf{r}) \epsilon_0 \mathbf{E}(\mathbf{r}, t)}{\partial t}$$

Take curl from both left and right sides of the first equation and then substituting second equation into the first one we obtain the following relation [5]:

$$-\nabla \times \nabla \times \mathbf{E}(\mathbf{r}, t) = \epsilon_r(\mathbf{r}) \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2}$$

The wave equation can be written in the following form:

$$\Delta \mathbf{E}(\mathbf{r}, t) + \frac{\epsilon_r(\mathbf{r})}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = 0.$$

We have the following dispersion relation

$$\cos\left(2\pi \frac{K}{g}\right) = \frac{1}{t_{12}t_{21}} \left[\cos\left(\pi \frac{\omega}{\omega_B}\right) - |r_{12}|^2 \cos\left(\pi \zeta \frac{\omega}{\omega_B}\right) \right]$$

Where,

$$g = 2\pi/\Lambda$$

$$\omega_B = c\pi/\Lambda$$

$$\Lambda = d_1 + d_2$$

$$n = (n_1 d_1 + n_2 d_2)/\Lambda$$

$$\zeta = (n_1 d_1 - n_2 d_2)/(n_1 d_1 + n_2 d_2)$$

$$t_{12}t_{21} = 4 n_1 n_2 / (n_1 + n_2)^2$$

$$|r_{12}|^2 = (n_2 - n_1)^2 / (n_1 + n_2)^2$$

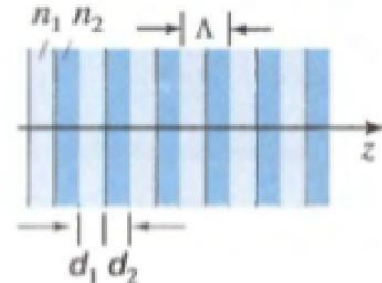


Fig.4 1D-Photonic crystal: alternating dielectric layers

The obtained linear system of equations is then solved by some of the standard methods such as Cramer's Method. As a result of the solution we have a set of amplitudes of forward and backward wave inside each penetrating layer.

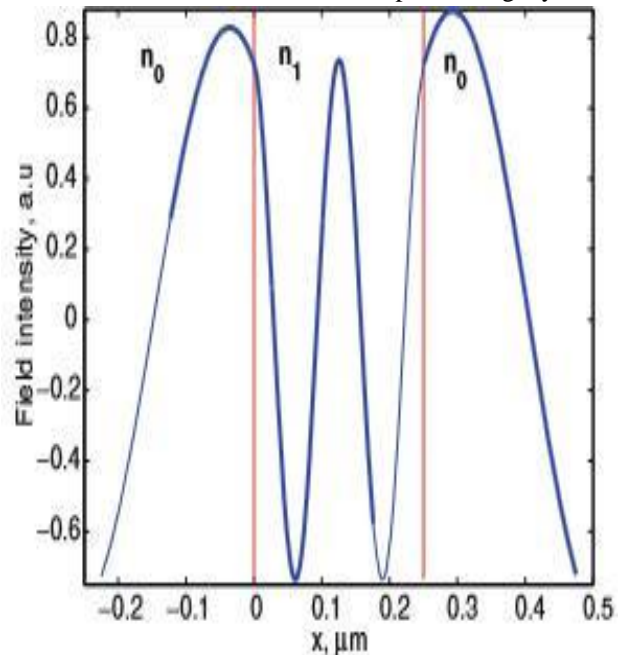


Fig.5 Field distribution inside 1D structure with one single negative layer

Brillouin zones [6] are readily defined in w-k space, even for situations that are only periodic in one space dimension as shown in Fig 6. All Brillouin zones have the same extent along the k-axis, since it is possible to add on a reciprocal lattice vector and obtain a solution that is formally indistinguishable. Finite difference time domain method is one of the major computational electromagnetic modelling methods.

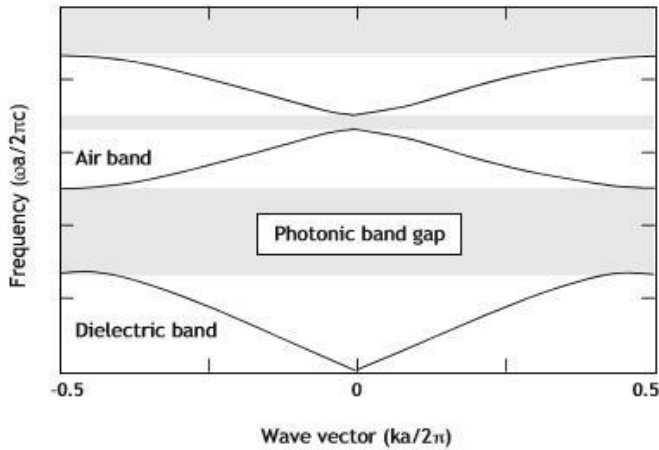


Fig.6 Schematic dispersion (w-k) diagram of large refractive-index contrast 1D periodic structure

FDTD technique [7] tackles problems by providing a full wave solution. FDTD, which is able to show transient evolution of interactions between electromagnetic wave and physical objects, not only has the advantage in dispersive and nonlinear material simulations, but also has the ability to model circuit elements including quiet a few semiconductor devices. We incorporated FDTD [8] after the analysis by PWE method for the band structure computation of 1D Photonic crystal. Latest techniques involve in the Finite difference time domain simulations of —metamaterials.

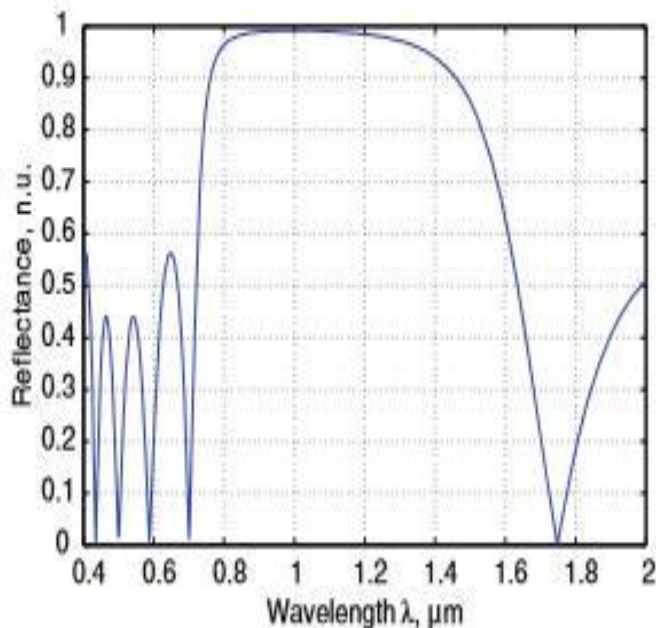


Fig.7 Typical reflectance spectrum of 1D Phc

We can observe that reflectance is quite different at different wavelengths. The Bragg wavelength here falls at a wavelength of around 1.20 micrometer which means that the maximum of reflectance can be observed at this wavelength respectively.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We analysed the program for photonic band structure computation of 1D photonic crystal. Unit cell consists of two layers. Structure is defined by the thickness of layers within the unit cell as well as their refractive index. Input parameters: thickness of layers and refractive index of each layer inside the unit cell. Output data: dependence of normalized frequency on wave vector (photonic band structure). Variables 11 and 12 contain thicknesses of each layer within the unit cell.

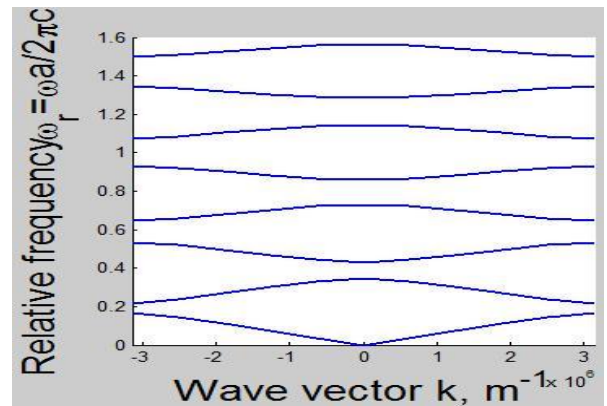


Fig.8 Band structure computation of 1D Phc for eps1= -1; eps2=9

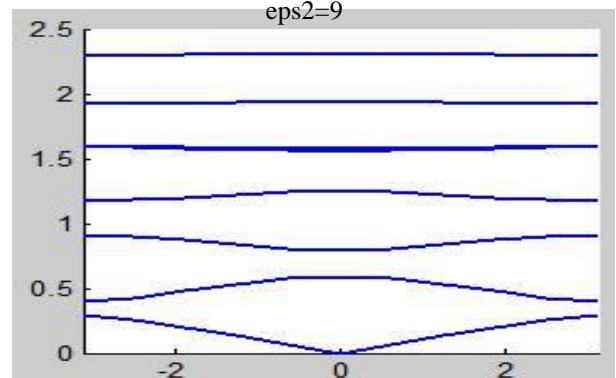


Fig.8 (a) Band structure computation of 1D Phc for eps1= -1; eps2=3

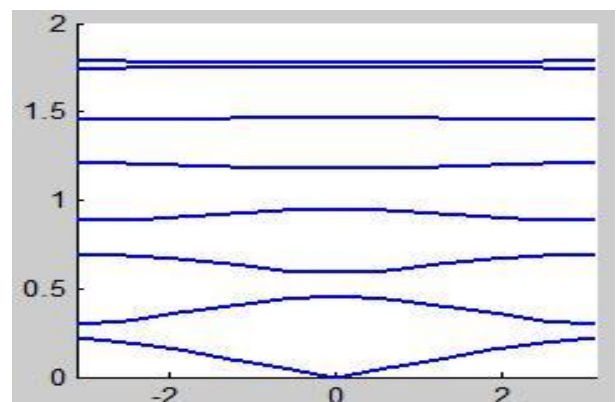


Fig.8 (b) Band structure computation of 1D Phc for eps1= -1; eps2=5

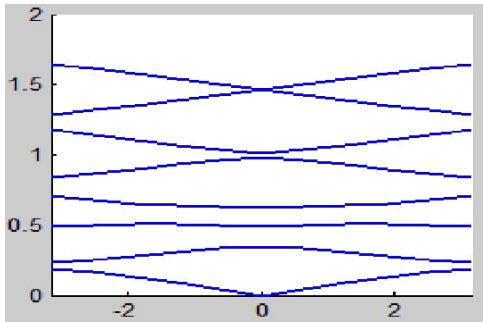


Fig.8 (c) Band structure computation of 1D Phc for $\epsilon_{p1}=-1$; $\epsilon_{p2}=7$

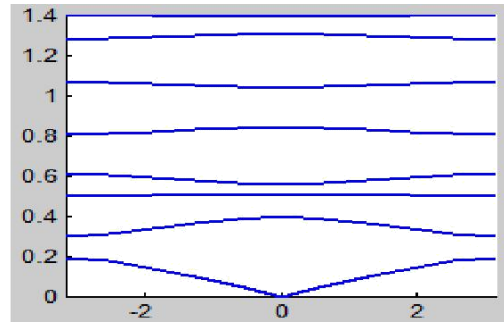


Fig.8 (g) Band structure computation of 1D Phc for $\epsilon_{p1}=-7$; $\epsilon_{p2}=7$

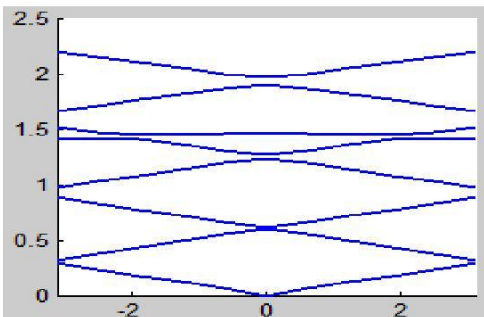


Fig.8 (d) Band structure computation of 1D Phc for $\epsilon_{p1}=-2$; $\epsilon_{p2}=3$

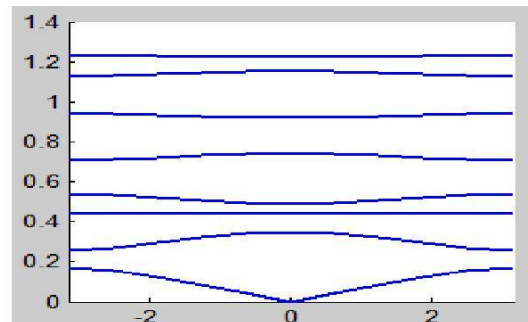


Fig.8 (h) Band structure computation of 1D Phc for $\epsilon_{p1}=-9$; $\epsilon_{p2}=9$

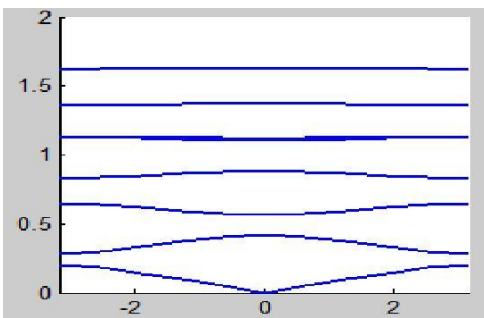


Fig.8 (e) Band structure computation of 1D Phc for $\epsilon_{p1}=-2$; $\epsilon_{p2}=6$

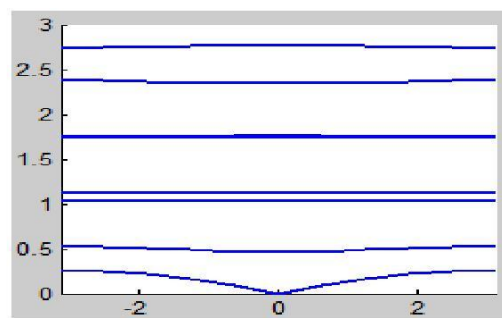


Fig.8 (i) Band structure computation of 1D Phc for $\epsilon_{p1}=-9$; $\epsilon_{p2}=1$

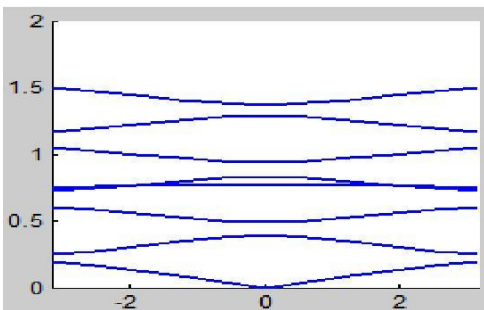


Fig.8 (f) Band structure computation of 1D Phc for $\epsilon_{p1}=-5$; $\epsilon_{p2}=7$

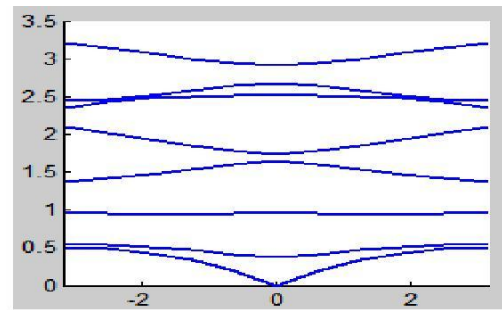


Fig.8 (j) Band structure computation of 1D Phc for $\epsilon_{p1}=-3$; $\epsilon_{p2}=1$

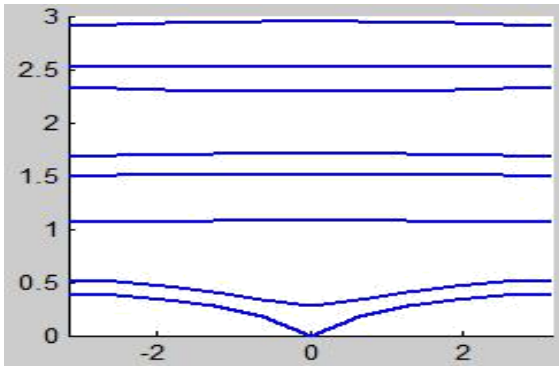


Fig.8 (k) Band structure computation of 1D Phc for eps1=-5; eps2=1

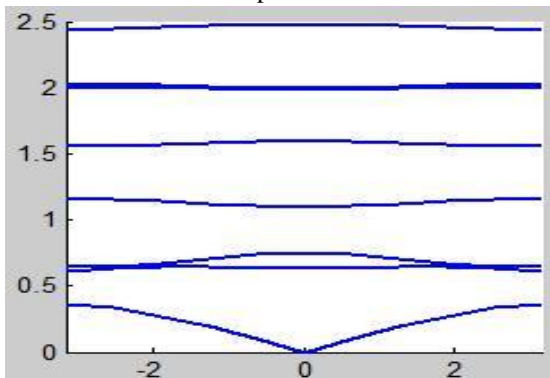


Fig.8 (l) Band structure computation of 1D Phc for eps1=-3; eps2=2

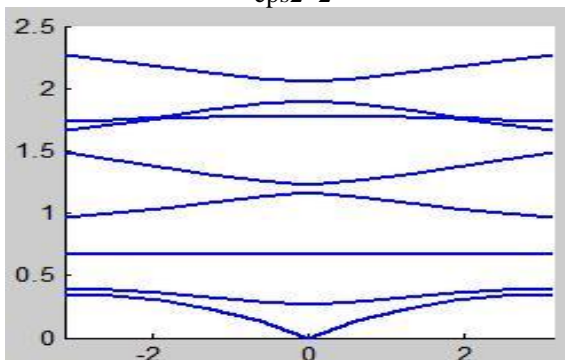


Fig.8 (m) Band structure computation of 1D Phc for eps1=-6; eps2=2

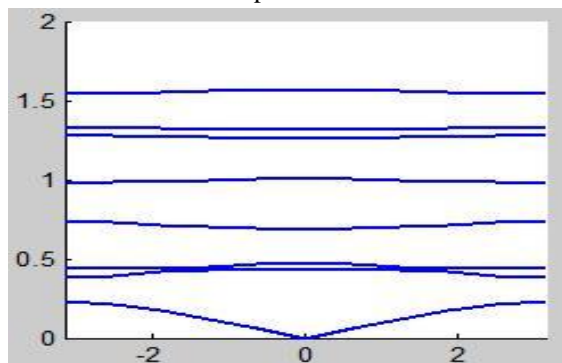


Fig.8 (n) Band structure computation of 1D Phc for eps1=-7; eps2=5

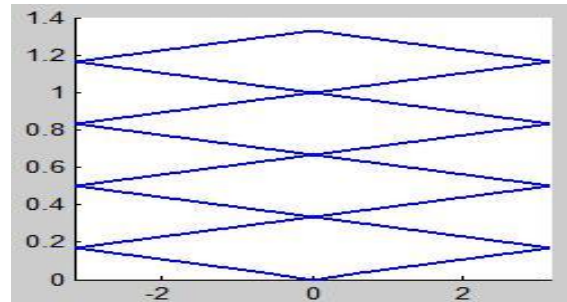


Fig.8 (o) Band structure computation of 1D Phc for eps1=9; eps2=9

The program computes the reflectance spectrum of the user-defined layered structure. The solution of the linear equations system is carried out by Cramer's method. For this reason, two matrices are composed. The first one contains coefficients at the unknowns of the system while the second one contains free terms. Input parameters: layers thicknesses and refractive indices, the range of the wavelengths for the spectrum computation. Output data: The plot of the layered structure reflectance spectrum within required range.

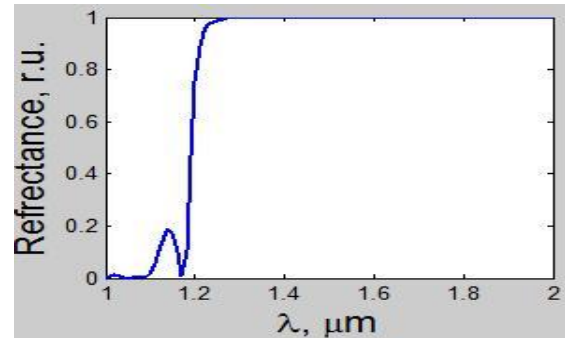


Fig.9 Typical reflectance spectrum of 1D Phc

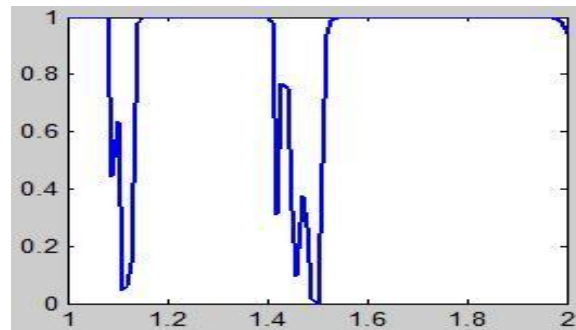


Fig.9 (a) Reflectance spectrum of 1D Phc for layer RI= [-1 9 -1 9 -1 9 -1];

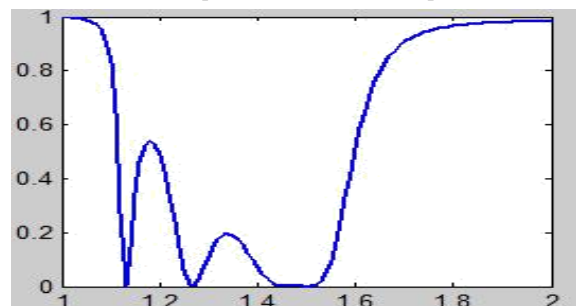


Fig.9 (b) Reflectance spectrum of 1D Phc for layer RI= [-1 3 -1 3 -1 3 -1];

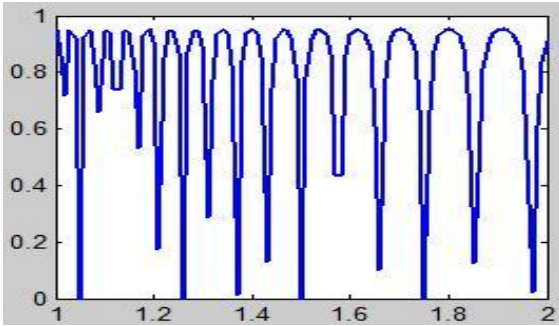


Fig.9 (c) Reflectance spectrum of 1D Phc for layer RI= [-9 9 -9 9 -9 9 -9 9];

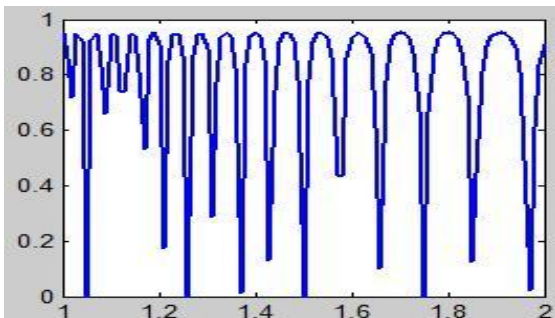


Fig.9 (d) Reflectance spectrum of 1D Phc for layer RI= [9 9 9 9 9 9 9 9];

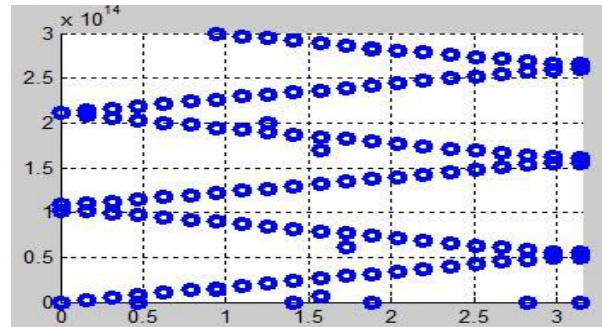


Fig.10 (c) Band structure computation of 1D Phc for eps1=6; eps2=9

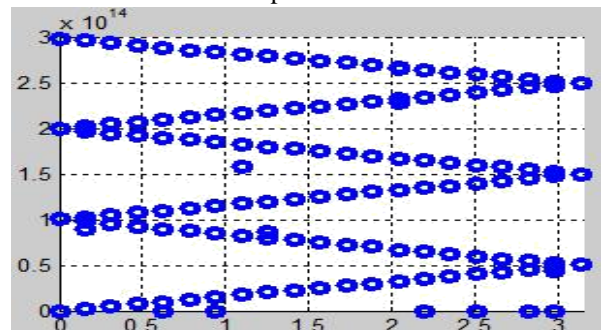


Fig.10 (d) Band structure computation of 1D Phc for eps1=9; eps2=9

We analysed the program for 1D PhC band structure computation by means of FDTD method [9]. Input data: permittivity distribution within unit cell and Output data: 1D PhC band structure. Where the variable a is lattice constant $a=1e-6$ and c is the light speed in vacuum $c=3e8$

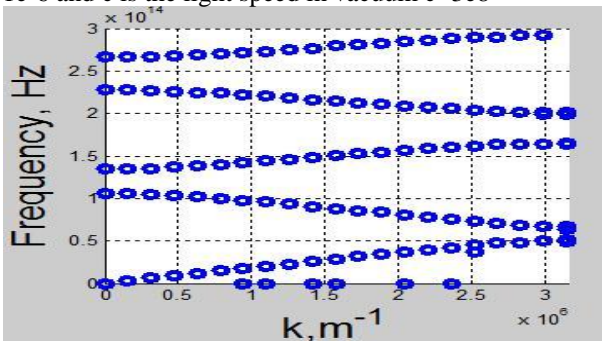


Fig.10 (a) Band structure computation of 1D Phc for eps1=1; eps2=9

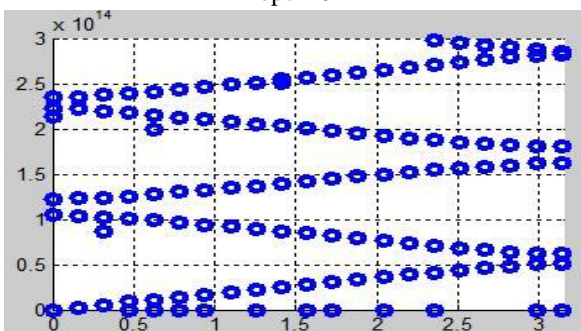


Fig.10 (b) Band structure computation of 1D Phc for eps1=3; eps2=9

V. SUMMARY AND FUTURE RESEARCH

Future work includes Finite difference time domain combined with SPICE and GPU (graphics processing units). Message passing interface (MPI) could be used to make parallel computing for FDTD and SPICE respectively. To overcome the GPU memory limitation of large scale structure simulation with FDTD, divided computational region can be utilized. Now this is a well promising area and will be useful especially for the ever-growing high frequency of the scientific and commercial applications. Due to such exuberant and wide range of properties exhibited by the photonic crystals they are widely used in nano-manipulation techniques, self-organization techniques, semiconductor process techniques, holographic techniques, quantum computing and communications etc.

VI. CONCLUSION

We infer from the graph that:

To increase the frequency range or to increase the band gap so that reflectance should occur for maximum frequency range. Ideally it should act as a mirror for the maximum range of frequencies and not for a particular range of frequency. So in both the methods either by PWE or FDTD our aim was always to increase the forbidden gap so that maximum reflections should occur inside the material itself. Since the main use of photonic crystals is as low-loss mirrors characterizing high reflectivity and low absorption loss per single reflection thus, for maximum reflectivity we should choose a crystal geometry that has the maximum photonic bandgap.

VII. ACKNOWLEDGMENT

I would like to express my sincere gratitude and respect to my guide who have made this work possible and helped me a lot during my entire research based learning program.

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