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MODEL ORDER REDUCTION OF TRANSFORMER COIL SECTION WITH AN IMPROVED POLE CLUSTERING

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Abstract: A dynamic time domain model lumped parameter model is used for modeling the frequency response of transformer. This paper presents a reduction technique which was the detailed lumped parameter linear transformer model order as a starting point and allows its reductions to any size of the model order specified by the user. In this proposed method, the reduced order denominator polynomial has been obtained using an improved clustering approach and its corresponding reduced order model was obtained through simple mathematical process. The proposed method is differ from some of the existing pole clustering technique by considering the distance of system poles from the first pole in the group clustering process. This process turnout better approximation in the reduction process. The result gained from this method is compared with the original system response. In this study, experimental model is taken as 10 section air cored coils of the transformer. It is obtain the reduced order transfer function, the response of the reduced model and original model is compared.
Index Terms: Transformer modeling, Model order reduction, Pole clustering, Dominant pole component.

I. INTRODUCTION

The behavior of large power transformers under transient conditions is of interest to both transformer former designer employs detailed electrical models to develop a reliable and cost effective transformer insulation structure. The power engineer models not only the transformer, but the system, in order to investigate the effects of power system transients. A comprehensive model will then include a high order network model with resistances (R), inductances (L) and capacitances (C). To perform system studies, utility engineers also need to represent the transformer in some detail. It is impractical to use the detailed model in system studies because of its size and the resultant computational burden. Finally, reduced order models are necessary for system studies. Reduced models are obtained either by calculations or by direct measurement on constructed transformers. Direct measurement technique only feasible for after construction of the transformer and It is not available for design stage[1-2].

Reduced order model [3] obtain from detailed model used for insulation design and appropriately combining of series and shunt elements. The reduction process is limited to linear models and cannot be used to eliminate a large proportion of the detailed models nodes without effecting the reduced models accuracy [4]. Terminal transformer model [5], composed of a synthesized RLC network, where the nodal admittance matrix approximates the nodal admittance matrix of the actualtrans former over the frequency range of interest. This methodology is appropriate only for linear models. Model order reduction techniques for time and frequency domain systems have been proposed by several researchers [6-9]. A large number of methods are available in the literature for order reduction of linear continuous systems in time as well as frequency domain. In this paper, a dynamic time domain model lumped parameter model is used for modeling the frequency response of transformer. Most of the model order reduction techniques are concerned with preserving stability and matching initial time moments between the full and reduced systems [10-11]. The stability of the system is preserved by obtaining the reduced order denominator polynomial based on selecting stable poles or using the properties of Routh table. To preserve the steady state characteristics it is usual either to solve the pade equations or invert a continued fraction, which yields the reduced numerator. Instead of using single method to derive the reduced model, now-a-days researchers prepare some mixed methods of model simplification for continuous time systems [12]. Stability preserving methods namely, (γ-δ) canonical expansion [13] and mihailov criterion has been combined with pade and factor division method to obtain the better approximation. Mihailov criterion is combined with the Cauer second form for reducing the order of the large scale SISO systems discussed in [14]. Few methods [15-21], uses the ISE as performance parameter which produces reduced order closer to given higher order system behavior. Recently evolutionary techniques such as Genetic algorithm, Particle Swarm Optimization are applied to obtain the better approximation [22]. In this method, denominator polynomial is obtaining by using any of the stability preserving criterions like stability equation method, mihailov stability criterion, routh approximation and etc. The clustering method proposed in this paper was slightly differ from some of the existing pole clustering technique by considering the distance of system poles from the first pole in the group clustering process. This process yields better approximation in the reduction process. The result obtained from this method is compared with the original system response. In this paper, experimental model is taken as reduced 10 section air cored transformer coils. Obtain the reduced order transfer function, the response of the reduced model and original model compared. Also, in this work the reduction model compared with balanced truncation methods and Hankel approximation method of continuous time state space models[23-26].

II. STATEMENT OF PROBLEM

The full order (10th order in this case) model poles and zeros are obtained from the process explained tabulated in Table I.

TABLE I. POLES AND ZEROS OF HIGHER ORDER (10 NODES) MODEL

	1	**2**	**3**	**4**	**5**
Poles	3.06	10.37	19.84	31.33	44.51
Zeros	7.39	16.8	28.45	41.92	56.54
	6	**7**	**8**	**9**	**10**
Poles	58.69	72.94	86.39	97.59	105.09
Zeros	71.44	85.45	97.14	104.97	--

Let the higher order transfer function of SISO linear time invariant system is in the form of Transformer Linear Section Model Order Reduction with an Improved Pole Clustering

$$G(S)=\frac{a_0+a_1s+a_2s^2+\dots\dots\dots+a_{m-1}s^{m-1}+a_ms^m}{b_0+b_1s+b_2s^2+\dots\dots\dots+b_{n-1}s^{n-1}+b_ns^n} \quad (1)$$

Where $m\le n$.

$$G(s)=\frac{\sum_{j=0}^{m}a_js^j}{\sum_{j=0}^{n}b_js^j}=\frac{N(s)}{D(s)} \quad (2)$$

The corresponding reduced order ('r') model should be in the form of

$$G_r(S)=\frac{d_0+d_1s+d_2s^2+\dots\dots\dots+d_{q-1}s^{q-1}+d_qs^q}{e_0+e_1s+e_2s^2+\dots\dots\dots+e_{r-1}s^{r-1}+e_rs^r} \quad (3)$$

Where $q\le r$

$$=\frac{\sum_{i=0}^{q}d_is^i}{\sum_{i=0}^{r}e_is^i}=\frac{N_r(s)}{D_r(s)} \quad (4)$$

The reduced order model have to be obtained so that it must retains all the characteristics of given higher order system.

III. PROPOSED METHOD

The proposed model order reduction method consists of two steps

Step 1: Retrieve the reduced order denominator polynomial with an improved pole clustering technique.

Calculate the 'n' number of poles from the given higher order system denominator polynomial. The number of cluster centres to be calculated is equal to the order of the reduced system. The poles are distributed in to the cluster centre for the calculation such that none of the repeated poles present in the same cluster centre. Minimum number of poles distributed per each cluster centre is at least one. There is no limitation for the maximum number poles per cluster centre. Let k number of poles available in a cluster centre: p_1, p_2, $p_3\dots p_k$. The poles are arranged in a manner such that $|p_1|<|p_2|<\dots\dots<|p_k|$. The cluster centre for the reduced order model can be obtained by using the following procedure.

- Let k number of poles available are $|p_1|<|p_2|<\dots\dots<|p_k|$,
- Set L=1,
- Find the pole cluster as , $C_L=[(-1/|p_1|+\sum_{i=2}^{k}-1/|p_i-p_1|)\div k]^{-1}$,
- Check for L=k. If yes, then the final cluster centre is $C_C=C_L$ and terminates the process. Otherwise proceed on to next step.
- Set L=L+1,
- The improved cluster centre from $C_L=[(-1/|p_1|+-1/|C_L|)\div 2]^{-1}$.
- Check for L=k. If no, then go to the step (5). Otherwise go the next step.
- Final cluster centre is $C_C=C_L$.

On calculating the cluster centre values, it is have following two cases as in.

Case (i). All the denominator poles are real:

The corresponding reduced order denominator polynomial can be obtained as,

$$D_r(s)=(s+C_{C1})(s+C_{C2})\dots\dots.(s+C_{Cr}) \quad (5)$$

Where $C_{C1},C_{C2}\dots\dots C_{Cr}$ are the improved cluster values required to obtain the reduced order denominator polynomial of order 'r'.

Case (ii). All the poles are complex:

Let 't' ($=k/2$) pairs of complex conjugate poles in a Lth cluster be,

$[(\sigma_1\pm j\omega_1), (\sigma_2\pm j\omega_2), (\sigma_3\pm j\omega_3)\dots(\sigma_t\pm j\omega_t)]$

Where,

$|\sigma_1|<|\sigma_2|<\dots.|\sigma_L|$. Apply the proposed algorithm individually for real and imaginary parts to obtain the respective improved cluster centres.

The improved cluster centre is in the form of

$\mu_j=A_j\pm jB_j$. (6)

Where, A_j and B_j is the improved pole cluster values obtained for real and imaginary parts respectively.

The corresponding reduced order denominator polynomial can be obtained as,

$$D_r(s)=(s+|\mu_1|)(s+|\mu_2|)\dots\dots(s+|\mu_j|) \quad (7)$$

Where, j=r.

Case (iii). If some poles are real and some poles are complex in nature, applying an improved clustering algorithm separately for real and complex terms. Finally obtained improved cluster centres are combined together to get the reduced order denominator polynomial.

Step 2: Retrieve the numerator polynomial of reduced system

Equate the given higher order system transfer function with the general form of reduced system transfer function. The reduced order denominator polynomial obtained from step 1 is utilized here to obtain the unknown values of reduced order system coefficients.

$$\frac{a_0+a_1s+a_2s^2+..+a_{m-1}s^{m-1}+a_ms^m}{b_0+b_1s+b_2s^2+..+b_{n-1}s^{n-1}+b_ns^n}=\frac{d_0+d_1s+d_2s^2+..+d_{q-1}s^{q-1}+d_qs^q}{e_0+e_1s+e_2s^2+..+e_{r-1}s^{r-1}+e_rs^r} \quad (8)$$

On cross multiplying the above equation and comparing the same powers of 's' on both sides, we get following (n+2) equations.

$$a_0e_0 = b_0d_0$$
$$a_0e_1 + a_1e_0 = b_0d_1 + b_1d_0$$
$$a_0e_2 + a_1e_1 + a_2e_0 = b_0d_2 + b_1d_1 + b_2d_0$$
$$: \quad :$$
$$a_me_r = b_nd_q$$

On solving above equations, it can find the unknown coefficients d_0, $d_1 \ldots d_q$. The reduced order numerator polynomial in the form of

$$N_r(s) = e_0 + e_1s + e_2s^2 + \ldots\ldots + e_{r-1}s^{r-1} + e_rs^r \quad (9)$$

IV. PROBLEM DESCRIPTION

Consider a 10^{th} order system with the transfer function of

$$G(S) = \frac{9.496e14 + 2.97e14s + 3.478e13s^2 + 2.062e12S^3 +}{9.4e15 + 5.344e15s + 7.13e14s^2 +} \cdot \frac{7.012e10S^4 + .. + 1.384e5s^7 + 574s^8 + s^9}{8.381e13s^3 + .. + 621.3s^9 + s^{10}}$$

(10)

By applying the improved pole clustering method, the corresponding reduced order denominator polynomial is,

$$Dr(s) = S^2 + 67.9629S + 209.7760 \quad (11)$$

By following the step 2 in the proposed method, the reduced order numerator polynomial can be obtained as,

$$N_r(s) = 1.4796S + 21.1915 \quad (12)$$

The reduced order model transfer function is,

$$G(s) = \frac{1.4796s + 21.1915}{s^2 + 67.9629s + .. + 209.7760} \quad (13)$$

The reduced order model is compared with some of the existing methods and an integral square error calculated between the original and reduced order system is calculated as

$$ISE = \sum_{i=0}^{n}[Y(t_i) - Y_r(t_i)]^2 \quad (14)$$

The Table II gives the comparison of the proposed method with balanced truncation three methods and Hankel Minimum Degree Approximation (HMDA) method. The proposed method concentrates on the distance between first pole and the remaining poles available in the pole clustering group. It is used to produce more dominant pole cluster value which will retain the properties of higher order system. The step response of original and reduced order model of proposed method is shown in Figure 1. The impulse response of original and reduced order model of proposed method is shown in Figure 2. The validity of the proposed method is evaluated by calculating an integral square error between original reduced order models. The Table II gives the comparison of transfer function for the proposed method with some of the existing methods. The Table III shows the closeness between the original and reduced models by calculating the values of Integral Square Error (ISE), Integral of Time multiplied by Absolute Error (ITAE) and Integral of Absolute Magnitude of the Error (IAE). The proposed method gives better result compared over some of the existing methods.

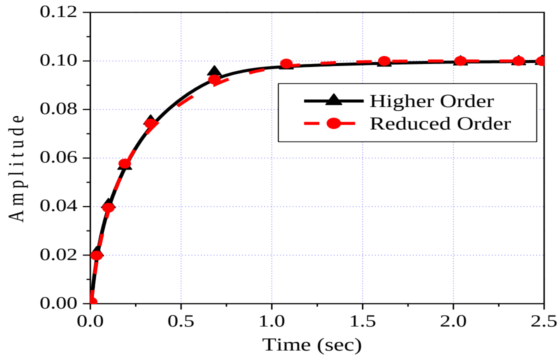

Fig. 1. Step response of original and reduced order model of proposed Method

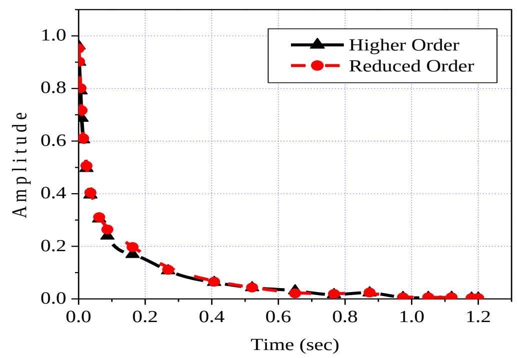

Fig. 2. Impulse respone of original and reduced order models of proposed method

TABLE II COMPARISON OF TRANSFER FUNCTION OF PROPOSED METHOD WITH THE EXISTING METHODS

Method	**Full Order Transfer function**	**Reduced Order Transfer function**
Balanced model truncation for normalized Co-prime factors	Equation(8)	$G_r(s) = \frac{0.9061s + 14.1908}{S^2 + 42.9120s + 142.7503}$
Balanced model truncation via square root method		$G_r(s) = \frac{0.9058s + 14.1741}{S^2 + 42.8800s + 142.5900}$

Balanced model truncation via Schur method		$G_r(s)=\frac{0.9060s+14.1768}{S^2+42.8800s+142.5479}$
Hankel Minimum Degree Approximation (MDA) Without Balancing		$G_r(s)=\frac{0.7891s+9.7653}{S^2+32.2130s+98.0404}$
Proposed Method		$G_r(s)=\frac{1.4796s+21.1915}{s^2+67.9629s+209.7760}$

TABLE IIIERROR VALUES BETWEEN HIGHER AND REDUCED ORDER MODELS

Method	ISE	ITAE	IAE
Balanced model truncation for normalized Co-prime factors	7.4349e-004	7.2623	0.4668
Balanced model truncation via square root method	7.4845e-004	7.2865	0.4683
Balanced model truncation via Schur method	7.0433e-004	7.0688	0.4544
Hankel Minimum Degree Approximation (MDA) Without Balancing	5.7993e-004	6.3871	0.4146
Proposed Method	8.2954e-005	0.0146	0.0227

V. CONCLUSION

In this section, it is evident that the transformer lumped parameter “n” order model is reduced to second order and retains all the characteristics of given higher order system. An improved pole clustering method along with simple mathematical procedure is proposed to obtain the reduced order system. The closeness between the original and approximated system is calculated by using ISE, ITAE and IAE as quality parameters. Stability of the reduced order model is assured if the given order higher orders system is stable.

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