GAME THEORY AND ITS APPLICATIONS

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Abstract: Situations involving interdependent decisions arise frequently in all walks of life. A few examples in which game theory and its fuzzification could come in handy are the following:

- ***Friends choosing where to go for dinner***
- ***Parents trying to get children to behave***
- ***Commuters deciding how to go to work***

All these situations call for strategic thinking-making use of available information's to devise the best plan to achieve one's objectives. The purpose of the present paper is to apply fuzzy game theory to extend this concept to interdependent decisions, in which the options being evaluated are functions of the player's choices.
Keywords: Fuzzification; fuzzy game theory; interdependent decisions; strategic thinking.

I. INTRODUCTION

Every day we go through while reading/watching in print and electronic media broadcasting reports containing mostly all branches of human knowledge, such as political, controversy, strike, armed conflict and violence by groups of people or countries, actions by various pressure groups to change social policy (now a day social justice), good decision about various types of policies and various branches of social justice, good attempt to control the financial market or all different types of scandals in different field. All the reports describe conflict of interest between groups of people. A theoretical model of such conflicts of interest is called "a game". Game theory consists of ways of analyzing the essential problems in the conflict of interest happening in real lives. This theory attempts to abstract those elements which are common and essential to many different competitive study situations and to study them by means of scientific methods. It is concerned with finding optional solutions or stable outcomes when various decisions may have conflicting objectives in their minds. The scope of game theory is rather broad and ambitious. It covers a much wider area such as national and international politics, management science, economics, business problems, education, sociology, psychology, optimization theory etc. Game theory is the area where quite different logical ideas can be expressed rigorously with minimum of mathematical sophistication. These ideas can be quickly comprehended, because there are manifestations in them in real life with which people are familiar. A game is determined by information, decision and goals, but human ideas and goals are fuzzy. Therefore, in a game, perfect informations, decisions and goals may not be feasible. The present work is concerned with introduction of game theory and its applications inclusive of mathematical preliminaries.

II. FUNDAMENTALS

(A). Combinatorial Topology and Simplex-
Combinatorial topology deals with those topological spaces which admit dissection into suitable regular pieces which are homomorphic to the interior of a triangle. Equivalently, we can formulate the concept of a standard space and then introduce the concept of building a space form such regular pieces or bricks. These bricks are called 'simplexes' which are generalizations of an interval (1- simplex), a triangle (2-simplex) or a tetrahedron (3-simplex).

(B). Convex hull-
A subset B of a Euclidian space Rn is said be 'convex' if given any two points x and y of B, the line segment joining x & y is entirely contained in B. Given any subset A of the convex hull of A is the intersection of all convex sets containing A and is also convex. The open P simplex < a0, a1, …..ap> determined by a set A = {a0, a1, ….. ak } of k + 1 point wise independent points of Rn is the convex hull of the set.

(C). Fuzzy Set Theory-
Basically a fuzzy set is a class in which there may be a continuum grade of membership in the class of long objects. Such fuzzy sets underline much of our ability to summarise, communicative and make decision under uncertainty or practical information.

III. FUZZY SET (ZADEH)

A fuzzy subset A of a universe of discourse X is characterized by a membership function,
$A: X \rightarrow [0, 1]$
Which associates with each element x of X, a member A(x) in the interval [0, 1] representing the grade of membership of x in A. In other words if A is a set in the usual sense then
$A(x) = 1$, if $x \in A$
$= 0$, if $x \notin A$

(A). Definition [complement of fuzzy sets]-
Let, A and B be two fuzzy subsets of X. Then we define
(i) $A \subset B \Leftrightarrow A(x) < B(x), \ \forall x \in X$
$A = B \Leftrightarrow A(x) = B(x), \ \forall x \in X$
(ii) A' is the complement of A
$\Leftrightarrow A'(x) = 1 - A(x), \ \forall x \in X$

III (B). Definition [Union of fuzzy sets]-
The union of two fuzzy sets A and B is a fuzzy set C written as
$C = A \cup B$ defined by
$C(x) = \max [A(x), B(x)]$,

$\forall\, x \in X$

Or, is abbreviated form

$C = A \vee B$

III(C). Definition [Intersection of fuzzy sets]-

The intersection of two fuzzy sets A and B is a fuzzy set Written as $C = A \cap B$ defined by

$C(x) = \min [A(x), B(x)],$

$\forall\, x \in X$

Or, in abbreviated form

$C = A \wedge B$

III (D). Definition [fuzzy relation]-

A fuzzy relation R from a set X to a set Y is a fuzzy subset of X x Y characterized by a membership function

$R : X \times Y \rightarrow [0, 1]$

For each $x \in X$, $y \in Y$, R (x, y) is referred as the strength of relation between x and y. If X =Y, then R is a fuzzy relation in X. The inverse of R is denote by R^{-1}, is a fuzzy relation from Y to X defined by,

$R^{-1}(x, y) = R'(x, y)$

If R and S are two fuzzy relations from X to Y and from Y to Z respectively, then composition of R and S, denoted by R o S is a fuzzy relation from X to Z defined by

$R \circ S(x, z) = V_y [R(x,y) \wedge S(y,z)],$

$x \in X,\ z \in Z$

(E). Definition [fuzzy pre-order relation]-

A Fuzzy relation R in X is a fuzzy pre-order relation iff,

(i) R is reflexive i.e.

$R(x,x) = 1,\ \forall\, x \in X$

(iii) R is transitive i.e

$R(x,z) \geq V_y[R(x,y) \wedge R(y,z)],\ \forall\, x,y,z \in X$

IV. NOTION OF GAME THEORY

Game theory is a collection of mathematical models formulated to study decision making in situation involving conflict and co-operation. We are confronted in our Daily life with complicated situations arising in decision making characteristic analysis of conflicting and co –operative situation led von Neumann to formulate game theory in 1928. A game is a quadruplet < N, □□, V, P> consisting of a set N of decision makes available to each player, a set V of outcomes each of which is a result of particular choice of strategies made by the players on a given play of game and a set P of payoff accorded to each player in each of the possible outcomes. Strategy is a complete plan for a given player that specifies which of the available choices he should make to each play. A player may choose his strategy probabilistically (by means of random device),a combination of such strategies called 'mixed strategies' determining a probability distribution of outcomes have also of the expected pay offs occurring to each player. In that case a player's decisions are guided by an attempt to maximize his expected pay off. Strategies that are not mixed are called 'pure'. A strategy which maximises or minimizes the possible average outcomes (gain or loss) of a given player in a number of plays of a game is called 'the optimal strategy of a player' constituting the solution of a game. Game of strategies can be classified with reference to the number of players. When n=2 it is called 'a two-person game', when n>2, it is referred to as 'an n- person game'. Some of the important topology of games are: Cooperative (in which choices and strategies may be co-ordinated), non cooperative (in which the choices and strategies may be made independently), constant sum (where sum of the pay offs to the player is the same in each outcome), non constant sum (where the interest of the players may partially coincide) and games with perfect information (in which each player is always informed about the whole history of the preceding play). The fundamental theorem of game theory proved by von Neumann asserts that every 2- person constant sum game with a finite number of strategies has a solution in the following sense: there exists available to each player at least one optimal strategy which many be pure or mixed. A player choosing such an optimal strategy can guarantee himself a certain minimum pay off. By definition of a constant sum game we mean that each player can keep the other's pay off down to the latter's guaranteed minimum. The resulting outcome of the game is equilibrium. A set of players that coordinate their strategies in order to promote their joint interest is called 'a coalition'. n-person game theory involves for the most part question related to the formation of coalition and the appointment of their gained payoffs among their members. Von Neumann- Morgenstern solution, advanced in the original treatise of game theory (1947) singles out certain set of appointments called 'imputations', each set being a solution of the given game. Shapley (1953) has advanced a method of singling out a single imputation of an n- person co-operative game, called 'the shapley value of game'. In a game there is the simple agreement that the loser pay the winner a certain amount whereas in the case of a stalemate there is no pay off. In this fashion, the pay off received by the n-players are in general uniquely determined as real valued functions of a chosen strategy n-tuple. These n functions are called 'the pay off function of the game'. During the Second World War there had been considerable activity in modeling decision situations, which involve one or more decision makers. Most of the military problems that can be modelled as games are of the two players zero -sum type, and these are the very ones for which game theory can suggest a specific solution. Thus at the end of the war, people were thinking of how to model decision situations, and there was a view that game theory had a successful (if secret) track record in military area.

V. CONCLUSION

Game theory is exciting because although the principles are simple, the applications are far reaching .It has opened up multiple new doors in the few decades it has been in practice. With it, all walks of life can now be modelled and accurately predicted. Through the continued study of applications of game theory we can come to better understand allied sciences and the intricate number system behind the field.

VI. ACKNOWLEDGMENT

The author would like to thank department of applied mathematics SSGI Junwani, Bhilai for providing the necessary motivation, guidance and software support.

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