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Application of Laplace Transform in differential Equations

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Abstract : *Some important terms in Laplace transform, electric circuit are studied. Examples related to electric circuit is also discuss in this paper.*

1. Intoduction

Laplace transform has an important role in differential equations corresponding initial and boundary value problems. The solution of differential equations involving functions of an impulsive type can be solved by the use of Laplace transform in a very effective method in a very efficient manner. Generally we can use following method to solve given differential equations.

(a) Firstly, the given differential equation is converted into an simple algebraic equation.

(b) Secondly simple algebraic equation can be solved by algebraic manipulations.

(c) The solution of simple algebraic equation is convert back to obtain the solution of the corresponding given differential equation.

In such way that, the Laplace transform method reduces the problem of solving differential equation to an algebraic problem. Important role of this method over the classimal method is that it solves IVP directly without finding its general solution and the values of the arbitrary constants.

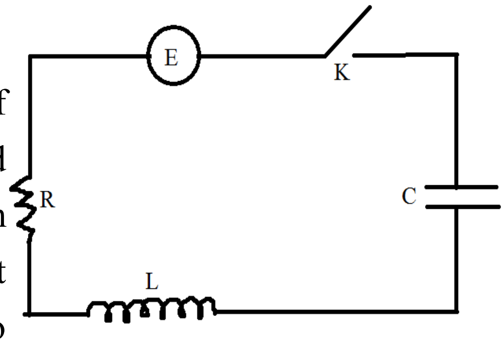
2. Some important terms in Laplace Transform :

(i) $L\left[\frac{dy}{dx}\right] = L\left[y^{'}\right] = s\overline{y} - y(0)$

(ii) $L\left[\frac{d^2y}{dx^2}\right] = L\left[y^{''}\right] = s^2\overline{y} - sy(0) - y^{'}(0)$

(iii) $L\left[\frac{d^3y}{dx^3}\right]=L\left[y'''\right]=s^3\bar{y}-s^2y(0)-sy'(0)-y''(0)$

2.1 Electric circuits



Consider a simple electrical circuit consist of Battery (emf) E, resistance K and Inductance L and Capacitance C are connected in series having switch or key K. When switch or key K is closed so that the circuit is completed, a charge Q will be flow to capacitor plates. The time rate of i.e. current = $i=\frac{dQ}{dt}$ is obtained then it can be measured in ampere. An important role of problem is to determine the charges on the capacitors and current as functions of time. Now we define potential drop across a circuit element using electrical law.

i) Voltage drop across a resister $=R_i=R\frac{dQ}{dt}$

ii) Voltage drop across an inductor $=L\frac{di}{dt}=L\frac{d^2Q}{dt^2}$

iii) Voltage drop across capacitor $=\frac{Q}{C}=\frac{1}{C}\int_0^t idt$

iv) Voltage drop across generator- voltage rise = -E

For L-C-R circuit by applying Kirchoff's law we obtain equation of Q

$$L\frac{d^2Q}{dt^2}+R\frac{dQ}{dt}+\frac{Q}{C}=E \tag{2.1}$$

This equation can be written in the form of current i is

$$L\frac{di}{dt^2}+Ri+\frac{1}{C}\int_0^t idt=E \tag{2.2}$$

Examples :

2.1.1 A constant electromotive force E is applied $t=0$ to an electrical circuit consisting of an inductance L, resistance R is connected in series then current (i) is given by

$$L\frac{di}{dt}-Ri=E(t)$$

If the switch is connected at $t=0$ and disconnected at $t=a$. Find the current *i* in terms of *t*.

Sol^n : Current *i* flows in circuit, given conditions are $i=0 \; at \; t=0$

$$E(t)=\begin{cases} E, & 0<t<a \\ 0, & t>a \end{cases}$$

Given equation is

$$L\frac{di}{dt}-Ri=E(t) \qquad (1)$$

Applying Laplace transform of (1), then

$$L\left[s\bar{i}-i(0)\right]-R\bar{i}=\int_0^\infty e^{-st}E(t)dt$$

$$L\left[s\bar{i}-(0)\right]-R\bar{i}=\int_0^\infty e^{-st}E(t)dt$$

$$(Ls-R)\bar{i}=\int_0^\infty e^{-st}E(t)dt$$

$$=\int_0^a e^{-st}Edt+\int_a^\infty e^{-st}Edt$$

$$=E\left[\frac{e^{-st}}{-s}\right]_0^a+E\left[\frac{e^{-st}}{-s}\right]_a^\infty$$

$$=\frac{E}{s}\left[1-e^{-as}\right]+0=\frac{E}{s}-\frac{E}{s}e^{-as}$$

$$\bar{i} = \frac{E}{s(Ls-R)} - \frac{Ee^{-as}}{s(Ls-R)}$$

taking inversion on both the side, then

$$i = L^{-1}\left[\frac{E}{s(Ls-R)}\right] - L^{-1}\left[\frac{Ee^{-as}}{s(Ls-R)}\right]$$

$$L^{-1}\left[\frac{E}{s(LR-R)}\right] = \frac{E}{L}L^{-1}\left[\frac{E}{s\left(s-\frac{R}{L}\right)}\right]$$

$$= \frac{E}{L}\frac{L}{R}L^{-1}\left[\frac{1}{s-R/L} - \frac{1}{s}\right]$$

$$= \frac{E}{R}\left[e^{\frac{R}{L}t} - 1\right]$$

and $L^{-1}\left[\frac{Ee^{-as}}{s(Ls-R)}\right] = \frac{E}{R}\left[e^{\frac{R}{L}(t-a)} - 1\right]u(t-a)$

(by second shifting theorem)

On substituting the values of the inverse transforms in (2), we get

$$i = \frac{E}{R}\left[e^{\frac{R}{L}t} - 1\right] - \frac{E}{R}\left[e^{\frac{R}{L}(t-a)} - 1\right]u(t-a)$$

$$= \frac{E}{R}\left[e^{\frac{R}{L}t} - 1\right] \quad for\ \ 0 < t < q\ \left[u(t-a) = 0\right]$$

$$i = \frac{E}{R}\left[e^{\frac{R}{L}t} - 1\right] - \frac{E}{R}\left[e^{\frac{R}{L}(t-a)} - 1\right] \quad for\ \ t > a, then\left[u(t-a) = 1\right]$$

$$= \frac{E}{R}\left[e^{\frac{R}{L}t} - e^{\frac{R}{L}(t-a)}\right]$$

$$= \frac{E}{R} e^{\frac{R}{L}t} \left[1 - e^{\frac{Ra}{L}} \right]$$

2.1.2 Solve $\frac{dx}{dt} + y = \sin t, \ \frac{dy}{dt} + x = \cos t \ \ given \ \ x(0) = 2 \ and \ \ y(0) = 0$

Sol[n] : Given $\frac{dx}{dt} + y = \sin t$

$$x' + y = \sin t \qquad (2.1)$$

$$\frac{dy}{dt} + x = \cos t$$

$$y^1 + x = \cos t \qquad (2.2)$$

Taking the Laplace transform of (1) and (2) then

$$\left[s\bar{x} - x(0) \right] + \bar{y} = \frac{1}{s^2 + 1}$$

$$s\bar{x} - 2 + \bar{y} = \frac{1}{s^2 + 1} \qquad (2.3)$$

From (2.2)

$$\left[s\bar{y} - y(0) \right] + \bar{x} = \cos t = \frac{s}{s^2 + 1}$$

$$s\bar{y} - 0 + \bar{x} = s\bar{y} + \bar{x} = \cos t \qquad (2.4)$$

From (2.3) and (2.4) then

$$s\bar{x} + \bar{y} = 2 + \sin t$$

$$s\bar{y} + \bar{x} = \cos t$$

2.2.3 Solve $\frac{dx}{dt} - y = 0 \ \ and \ \ \frac{dy}{dt} + x = 0$

under the condition $x(0) = 1, y(0) = 0$

Sol[n] : We have

$$\frac{dx}{dt} - y = 0, \ i.e. \ x' - y = 0 \qquad (2.1)$$

$$\frac{dy}{dt} + x = 0, \ i.e. \ y' + x = 0 \qquad (2.2)$$

$$x(0) = 1. \ \ y(0) = 0$$

By applying Laplace transform of (1), (2) then

$\left[s\bar{x} - x(0)\right] - \bar{y} = 0$

$s\bar{x} - 1 - \bar{y} = 0$

$$s\bar{x} - 1 = \bar{y} \qquad (2.3)$$

and $\left[s\bar{y} - y(0)\right] + \bar{x} = 0$

$s\bar{y} - 0 + \bar{x} = 0$

$$\bar{x} = s\bar{y} \qquad (2.4)$$

Substituting (4) in (3) then

$s(s\bar{y} - 1) = \bar{y}$

$s^2\bar{y} - 1 = \bar{y}$

$s^2\bar{y} - y = 1$

$\bar{y}(s^2 - 1) = 1$

$$\bar{y} = \frac{1}{(s^2 - 1)} \qquad (2.5)$$

substituting these value in (4) we get

$$\bar{x} = s\bar{y} = \frac{s}{s^2 - 1} \qquad (2.6)$$

Applying inversion for equation (5) and (6) then

$x = L^{-1}\left(\frac{s}{s^2 - 1}\right) \; and \;\; y = L^{-1}\left(\frac{1}{s^2 - 1}\right)$

$x = \cosh t \qquad and \;\; y = \sinh t$

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