

GENERAL ANALYSIS OF INTERDEPENDENCY OF FRICTION FACTOR OF A CIRCULAR PIPE WITH REYNOLDS NUMBER OF THE FLOWING FLUID & RELATIVE ROUGHNESS PARAMETER OF THE PIPE MATERIAL

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Abstract: The loss of head due to friction in a flow through pipes may be correctly predicted if only the friction factor can be evaluated accurately. The loss of head due to friction may be evaluated using Darcy Weisbach Equation.

$$h = \frac{fLV^2}{2gD}$$

Here h is loss of head due to friction, f is friction factor, L is characteristic length of pipe, V is velocity of flow of fluid, g is acceleration due to gravity, D is diameter of pipe. By dimensional analysis, it can be shown that friction factor f depends on Reynolds number (VD/ν) and the ratio of (ϵ/D) .

$$\text{Thus, } f = \left[\left(\frac{VD}{\nu} \right), \left(\frac{\epsilon}{D} \right) \right] \dots\dots\dots (a)$$

Where ϵ is the average height of pipe wall roughness protrusions, ν is the kinematic viscosity of the fluid flowing in pipe. The term (ϵ/D) is commonly known as Relative Surface Roughness. Equation (a) is the general eq. valid for laminar as well as turbulent flow through pipes. This paper is based on the dependency of friction factor on the relative roughness of pipe categorized as hydrodynamically smooth and rough pipes and the Reynolds number of the fluid flowing through pipe.

I. INTRODUCTION

1. Friction Factor For Laminar Flow:

The friction factor for laminar flow depends only on Reynolds number and is independent of relative roughness. The f for laminar flow is given by

$$f = \left(\frac{64}{Re} \right) \dots\dots\dots (b)$$

It has been verified experimentally for Reynolds number equal to 2000, which is limit for the flow to be laminar in circular pipes

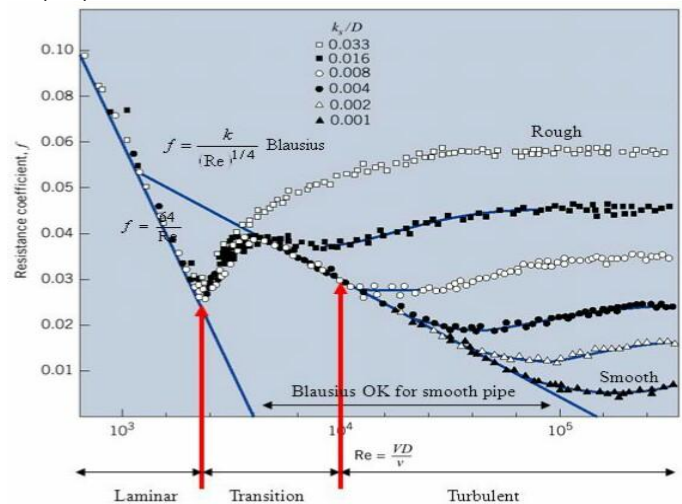
Pipe Material	Absolute Roughness, ϵ	
	$\times 10^{-6}$ feet	microns
drawn brass or copper	5	1.5
commercial steel	150	45
wrought iron	150	45
asphalted cast iron	400	120
galvanized iron	500	150
cast iron	850	260
wood stave	600 - 3000	200 - 900
concrete	1000 - 10,000	300 - 3000
riveted steel	3000 - 30,000	900 - 9000

2. Friction Factor For Turbulent Flow :

For Reynolds number between 2000 and 4000, there exists a zone of transition from laminar and turbulent flow. In transition zone, however, Re does not have any specific interdependency with f . In fully developed turbulent flow, f depends on both Re and (ϵ/D) , depending on whether the boundary is hydrodynamically smooth or rough or in transition zone. The relationship for the friction factor f for the different boundaries is as discussed below.

Variation in friction factor for turbulent flow in smooth pipes For turbulent flow in smooth flow in a circular pipe depends only on the Reynolds number and it is independent of the relative roughness (ϵ / D) because in smooth pipes the roughness protrusions are submerged in the laminar sublayer thus will have no effect on the friction factor. In 1911, Blasius studied the experimental data of Saph and Schoder and developed the following empirical equation for turbulent flow with smooth boundaries

$$f = \frac{0.316}{(Re^{1/4})} \dots\dots\dots (c)$$



Equation (c) is plotted as shown in figure using $\log f$ and $\log Re$ as coordinates. On the same plot Nikuradse's experimental data for smooth pipe is also plotted.

Observations: For Reynolds number varying from 4×10^3 to 10^5 the experimental data follow the equation (c) closely for Re number greater than 105 there is apparent deviation of the experimental points from the straight lines. Thus it was observed that Blasius Equation is valid for hydrodynamically smooth pipes for Re number from 4×10^3 to 105. The friction factor f for Re number greater than 105 may be obtained

from the expression derived from the logarithmic law of velocity distribution for smooth pipes. For turbulent flow in smooth pipes the expression for mean velocity is given as follows.

$$\frac{V}{V^*} = 5.75 \log\left(\frac{V^*R}{\nu}\right) + 1.75 \quad \dots\dots\dots (d)$$

Here R is the radius of pipe. Since shear velocity $V^* = V \sqrt{\frac{f}{8}}$, so substituting the value of V^* in equation (d) we get

$$\frac{V}{V \sqrt{(f/8)^{1/2}} R} = 5.75 \log\left(\frac{V \sqrt{(f/8)^{1/2}} R}{\nu}\right) + 1.75$$

Which may be simplified and rearranged in the following form $\frac{1}{\sqrt{f}} = 2.03 \log\left(\frac{VD}{\nu} \sqrt{f}\right) - 0.91$

$$\frac{1}{\sqrt{f}} = 2.03 \log(Re \sqrt{f}) - 0.91 \quad \dots\dots\dots (e)$$

However on the basis of Nikuradse's experimental data for turbulent flow in smooth pipes, it was observed that the trend follows the following equation instead of equation (e)

$$\frac{1}{\sqrt{f}} = 2.0 \log(Re \sqrt{f}) - 0.8 \quad \dots\dots\dots (f)$$

Equation (f) is known as 'Karman- Prandtl resistance equation' for turbulent flow in smooth pipes.

From Nikuradse's experimental equation (e) has been found valid for Re number 5×10^4 to a Re number as high as 4×10^7 . To calculate the friction factor directly the following equation is used

$$f = 0.0032 + \frac{0.221}{(Re)^{0.237}} \quad \dots\dots\dots (g)$$

Variation of Friction Factor for Turbulent Flow in Rough Pipes

The friction factor for Turbulent flow in Rough pipes is a function of Relative roughness only and is independent of the Reynolds number. The expression however, could be obtained similarly as that for the smooth pipe Turbulent flow. The mean velocity for turbulent flow in rough pipe can be expressed as following

$$\frac{V}{V^*} = 5.75 \log\left(\frac{R}{\epsilon}\right) + 4.75 \quad \dots\dots\dots (a)$$

Since we know $V^* = V \sqrt{f/8}$, so by substituting this value in equation (a)

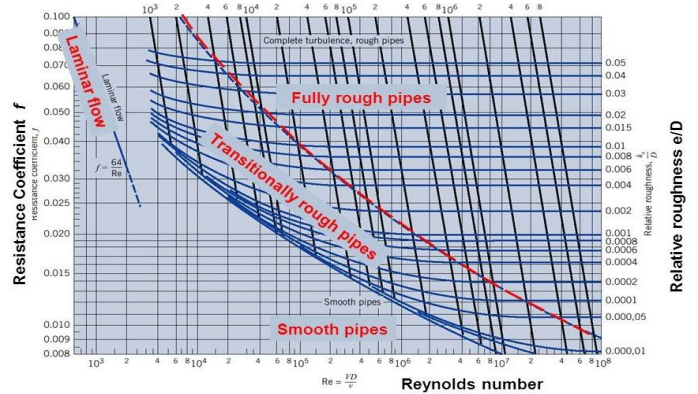
$$\frac{V}{V \sqrt{f/8}} = 5.75 \log\left(\frac{R}{\epsilon}\right) + 4.75 \quad \dots\dots\dots (b)$$

Which may be simplified as $\frac{1}{\sqrt{f}} = 2.03 \log\left(\frac{R}{\epsilon}\right) + 1.68 \dots\dots\dots (c)$

Again Nikuradse's experimental data on turbulent flow through pipes has shown that the experimental results instead of following the trend of equation (b)

$$\frac{1}{\sqrt{f}} = 2.0 \log\left(\frac{R}{\epsilon}\right) + 1.74 \quad \dots\dots\dots (d)$$

This is known as 'Karman - Prandtl Resistance Equation For Turbulent Flow in Rough pipes' while comparing the two equations (c) and (d) there is only slight difference in the values of numerical constants.



II. CONCLUSION

From the above discussion it can be concluded as follows
 Friction Factor For Smooth Pipes : It depends only on the Reynolds number (VD/ν) and is independent of the relative roughness (ϵ/D) of the pipe because in hydrodynamically smooth surface, the roughness protrusions are submerged in laminar sublayer and hence will have no effect on the friction factor as shown by derivation above.

Also the value of friction factor for Turbulent flow in hydrodynamically smooth pipes can be calculated from 'Nikuradse's experimental measurement equation' and also from 'Karman - Prandtl Resistance Equation' for Turbulent flow in smooth pipes.

Friction factor For Rough Pipes :It depends only on surface roughness parameter and is independent of Reynolds number. In other words, for any given value of the surface wall parameter (R/ϵ) the friction factor f has a constant value, which is indicated by various (R/ϵ) curves.

Also the figure indicates that greater the value of surface parameter (R/ϵ), larger is the value of Reynolds number Re at which the pipe, which originally behaves as smooth pipe began to follow the pattern of Rough pipe.

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