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COMPARISON OF MECHANICAL BEHAVIOUR OF CIRCULAR STEPPED BEAM USING FEM & STAAD.PRO

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Abstract:** In the present world, the increasing demand of structurally efficient and significantly higher strength to weight ratio structures is mostly served by stepped beam. These structural elements made up of steel and aluminum material which generally loading is applied, and beam elements located at discrete spacing in one or both directions. The present work deals with the structural behaviour of a beam under static uniform loads. Firstly, we will consider a geometrically nonlinear beam problem by analyzing the large deflections of a beam of linear elastic material, under the action of transverse load along its length. Under the action of these external loads, the beam deflects into a curve called the elastic curve. Firstly, the relationship between the beam deflection and the loads would be established using STAAD.PRO software and then the results would be extended to perform analysis on beam .The simulation analysis is completed with a numerical analysis of the system using the STAAD.PRO program, a comprehensive finite element package, which enables to solve the nonlinear differential equation. STAND.PRO provides a rich graphics capability that can be used to display results of analysis on a high-resolution graphics workstation.
***Keywords:** FEM (Finite Element Method), Stepped beam, Stress & elongation analysis, STAAD.PRO etc*

I. INTRODUCTION

The stepped beams can be found in many engineering applications in shafts, antennae, rotor blades, gun barrels, slender structures, and so forth. The changes in the cross-sectional areas and the load distribution generate discontinuities in deriving the deflection equation of a stepped beam. In the present world, the demand for structures with high stiffness is increasing day by day. One of the ways to deal with it is by using stiffeners. Countless mechanical structures are composed of stepped beam.

A series of linearly varying beam cross-sections can be created using the Variable Cross-Section Wizard. Each beam element has a different cross-section dimension based on interpolating the two user-specified end dimensions. The wizard creates a stepped beam that approximates a tapered beam. Any user-defined shape (rectangle, wide-flange beam, channel, pipe, and so on) whose dimensions can be entered in the normal cross-section library dialog box can be used for the variable cross-section.

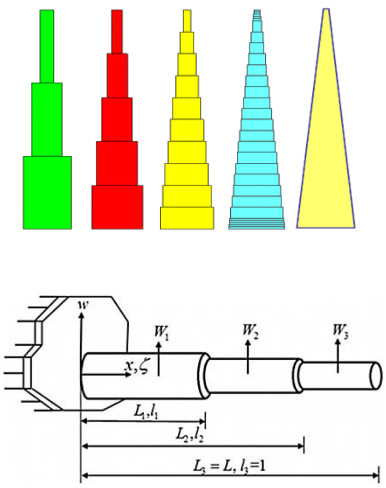

Fig 1.1: Different Types of Stepped Beam

A. Applications of Stepped beam

Some applications of stepped beam widely used in different places:

- BRIDGES: As mentioned in the introduction, the Firth of Forth Bridge is an excellent example of using the Stepped beam shape for structural applications in bridges.
- BUILDINGS, HALLS, etc.: In buildings and halls, hollow sections are mainly used for beam and lattice girders or space frames for roofs. In modern architecture they are also used for other structural or architectural reasons, e.g. facades
- TOWERS AND MASTS: Considering wind loading, corrosion protection and architectural appearance, however, in many countries, electrical transmission towers are made of angle sections with simple bolted connections.

II. LITERATURE REVIEW

The beam with variable cross-section is often modelled by a large number of small uniform elements, replacing the continuous changes with a step law. This scheme is accurate for a stepped beam but approximated for a beam with continuously changed cross-section. Although in this way, it is always possible to reduce errors as much as desired and obtain acceptable results by refining meshes, the modelling and computational efforts can become excessive. The boundary element methods for static torsion and torsional vibration analyses of bars with variable cross-section were developed by Sapountzakis and co-workers [7, 8]. The static responses of curved beam with variable cross-section were studied [5], in which the stiffness matrix and the equivalent nodal loads of the curved beam element were presented. The Carrera Unified Formulation was derived by Carrera and co-workers [2, 6], and under that framework, they presented a method to analyze beams with arbitrary cross-sectional geometries. Firouz-Abadi et al. [5] presented a Wentzel, Kramers, Brillouin approximation-based analytical solution to free transverse vibration of a class of varied cross-section beams. The use of exact displacement interpolation functions to solve varied cross-section beam problems is a straightforward way; however, they [9, 11].

III. METHODOLOGY

The analysis is done using Finite Element Method and the simulation is done using STAAD.PRO. The advantage of using the FEM methodology is that unlimited number of stiffeners can be added to the model, which can be placed at any direction inside the plate element [4]. The formulation accepts eccentric and concentric stiffeners of different cross-sections.

A. Finite Element Method (FEM)

The finite element method (FEM) (its practical application often known as finite element analysis (FEA)) is a numerical technique for finding approximate solutions to partial differential equations (PDE) and their systems, as well as (less often) integral equations. In simple terms, FEM has an in built algorithm which divides very large problems (in terms of complexity) into small elements which can be solved in relation to each other. FEM solves the equations using the Galerkin method with polynomial approximation functions. The solution is obtained by eliminating the spatial derivatives from the partial differential equation. This approximates the PDE with

- A system of algebraic equations for steady state problems
- A system of ordinary differential equations for transient problems.

These equation systems are linear if the corresponding PDE is linear and vice versa. Algebraic equation systems are solved using numerical linear algebra methods.

B. STAAD.Pro

STAAD or (STAAD.Pro® V8i) is a structural analysis and design computer program originally developed by Research Engineers International in Yorba Linda, CA. In late 2005, Research Engineers International was bought by Bentley Systems. It is the World's #1 Structural Analysis and Design Software. The analysis is done in a numerical way by the STAND.PRO program, a finite element package, which enables us to solve the linear and the nonlinear PDE's and thus the modulus of elasticity of the beam material is obtained . STAND.PRO is modeling and analysis software which helps in the modeling and analysis of required models, a FEM tool. It is used to analyze complex problems in mechanical structures, thermal processes, electrical fields, magnetics, and computational fluid dynamics. STAND.PRO provides a rich graphics environment, which is used to display results of analysis that re performed.

STAAD.Pro is a comprehensive and integrated finite element analysis and design offering, including a state-of-the-art user interface, visualization tools, and international design codes. It is capable of analyzing any structure exposed to static loading, a dynamic response, soil-structure interaction, wind, earthquake, and moving loads. STAAD.Pro V8i is the premier FEM analysis and design tool for any type of project including towers, culverts, plants, bridges, stadiums, and marine structures.

3.2 Analysis of stepped beam using Finite Element Method (FEM)

Calculation with the Help of FEM Method:-

Case-I:- Determine the nodal displacements at node 2 stresses in each material and support reactions in the bar shown in Fig. 3.1 due to applied force $P = 400 \times 10^3$N and temperature rise of 300°C. Given:

$A_1 = 2400\,mm^2, A_2 = 1200\,mm^2$

$l_1 = 300\,mm, l_2 = 400mm$

$E_1 = 0.7 \times 10^5 N/mm^2, E_2 = 2 \times 10^5 N/mm^2$

And $\alpha_1 = 22 \times 10^{-6}/C^0, \alpha_2 = 12 \times 10^{-6}/C^0$

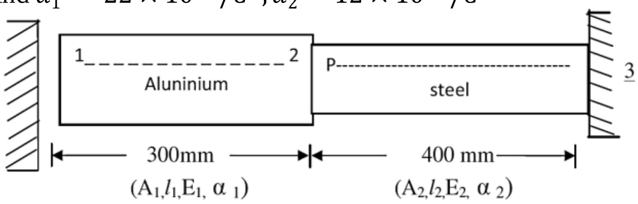

Fig 3.1 Stepped beam at case 1

Solution:

$$[K]_e = \frac{E_1 A_1}{l1} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \frac{0.7 \times 10^5 \times 2000}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^3 \begin{bmatrix} 560 & -560 \\ -560 & 560 \end{bmatrix}$$

$$[K]_2 = \frac{0.7 \times 10^5 \times 2000}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 10^3 \begin{bmatrix} 600 & -600 \\ -600 & 600 \end{bmatrix}$$

$$\therefore [K] = 10^3 \begin{bmatrix} 560 & -560 & 0 \\ -560 & 560+600 & -600 \\ & -600 & 600 \end{bmatrix}$$

$$= 10^3 \begin{bmatrix} 560 & -560 & 0 \\ -560 & 1160 & -600 \\ 0 & -600 & 600 \end{bmatrix}$$

Nodal force vector

Due to temperature changes

$$\{F_{eT}\}_1 = 0.7x10^5x2400x22x10^{-6}x30 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$$

$$= \begin{array}{c} \begin{Bmatrix} -110880 \\ 110880 \end{Bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \end{array}$$

$$\{F_{eT}\}_2 = 2x10^5x1200x12x10^{-6}x30 \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} -86400 \\ 86400 \end{Bmatrix} \begin{array}{c} 21 \\ 3 \end{array}$$

$\therefore$

$$\{F_{eT}\} = \begin{Bmatrix} -110880 \\ 110880 - 86400 \\ 86400 \end{Bmatrix} = \begin{Bmatrix} -110880 \\ 24480 \\ 86400 \end{Bmatrix}$$

Due to applied forces

$$\{F\} = \begin{Bmatrix} 0 \\ 400000 \\ 0 \end{Bmatrix}$$

$\therefore$Load vector due to applied loads and temperature effect is

$$\{F\} = \begin{Bmatrix} -110880 + 0 \\ 244480 + 400000 \\ 086400 + 0 \end{Bmatrix} = 10^3 \begin{Bmatrix} -110.88 \\ 424.48 \\ 86.10 \end{Bmatrix}$$

The boundary conditions are $\delta_1 = \delta_3 = 0$

$\therefore$The equation reduced to

$1160\delta_2 = 424.48$

i.e.δ_2 = 0.36593 mm

σ = Ee [B] { }e – Ee, αe, ΔT

$$\therefore \sigma_1 = 0.7\,\mathrm{x}10^5\mathrm{x}\,\frac{1}{300}[-1 \quad 1]\begin{Bmatrix} 0 \\ 0.36593 \end{Bmatrix} - 0.7\,\mathrm{x}\,10^5\ \mathrm{x}\,22\,\mathrm{x}\,10^{-6}\mathrm{x}\,30$$

$\sigma_1 = 39.18\ N/mm^2$

$$\sigma_2 = 2\,\mathrm{x}10^5\mathrm{x}\,\frac{1}{400}[-1 \quad 1]\begin{Bmatrix} 0.36593 \\ 0 \end{Bmatrix} - 2\,\mathrm{x}\,10^5\ \mathrm{x}\,12\,\mathrm{x}\,10^{-6}\mathrm{x}\,30$$

$\sigma_2 = 50.965\ N/mm^2$

$$R_1 = \{K_{11} \quad K_{12} \quad K_{13}\}\begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \end{Bmatrix} - f$$

$$= 10^3[560 \quad -560 \quad 0]\begin{Bmatrix} 0 \\ 0.36593 \\ 0 \end{Bmatrix} + 110080$$

$$R_1 = -94.041\ kN$$

$$R_3 = 10^3[0 \quad -600 \quad 600]\begin{Bmatrix} 0 \\ 0.36593 \\ 0 \end{Bmatrix} - 86400$$

$R_3 = -305.9\ kN$

$$[\text{Check}: \sum H = 0 \rightarrow -94040 + 400000 - 305959 = 0]$$

Case-II: A stepped bimetallic bar, made of aluminum ($E = 70 \times 10^3 N/\,mm^2$) and steel ($E = 200 \times 10^3 N/\,mm^2$) , is subjected to the axial force of 5000 N, as shown in Figure 3.2 It is attached to the rigid wall at node. 1 Using the matrix analysis method determined:

- The nodal displacements; and
- The reaction force at support.
- Also calculate stresses.

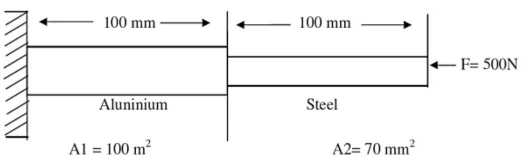

Fig 3.2 Stepped beam at case 2

Solution:

$A_1 = 100\,mm^2, A_2 = 70\ mm^2$

$l_1 = 100\ mm, l_2 = 100mm$

$E_1 = 70 \times 10^3 N/\,mm^2, E_2 = 200 \times 10^3 N/\,mm^2$

Solve similarly to case 1 then we get,

Nodal displacements

$\delta_2 = 0.07143\ mm$

$\delta_3 = 0.1071\ mm$

Reaction forces at support

R= 5 kN

Stresses

$\sigma_1 = 65.258\ N/mm^2$

$\sigma_1 = 46.589\ N/mm^2$

Case-III: Consider the bar shown in figure below. An axial load of 15 KN is applied as shown in figure 3.3:

- Determine the displacement at each node.
- Determine the stress in each element and the reaction at the fixed ends.

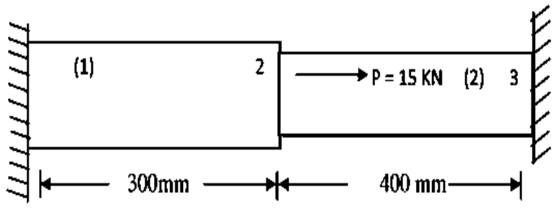

Fig 3.3 Stepped beam at case 3

The area cross-section and Young's modulus are given in the followings:

$A_l = 600\,mm^2, A_{br} = 300\ mm^2$

$E_l = 70 \times 10^3 N/\,mm^2, E_{br} = 83 \times 10^3 N/\,mm^2$

Solution: Solve the above question similar to above cases, also we calculated:

Determine the displacement at each node.

$\delta = 0.0742$mm

Determine the stress in each element and the reaction at the fixed ends.

$\sigma_1 = 16.8\ N/mm^2$

$\sigma_2 = -15.39\ N/mm^2$And

$R_1 = -10.388\ kN$

$R_3 = -4.618\ kN$

3.3 Analysis using STAAD.Pro

Calculation with the Help of software Method:-

CASE 1:- Determine the nodal displacements at node 2 stresses in each material and support reactions in the bar shown in Figure 3.4 due to applied force $P = 400 \times 10^3$N and temperature rise of 300°C. Given:

$A_1 = 2400\,mm^2, A_2 = 1200\ mm^2$

$l_1 = 300\ mm, l_2 = 400mm$

$E_1 = 0.7 \times 10^5 N/\,mm^2, E_2 = 2 \times 10^5 N/\,mm^2$

And $\alpha_1 = 22 \times 10^{-6}/C^0, \alpha_2 = 12 \times 10^{-6}/C^0$

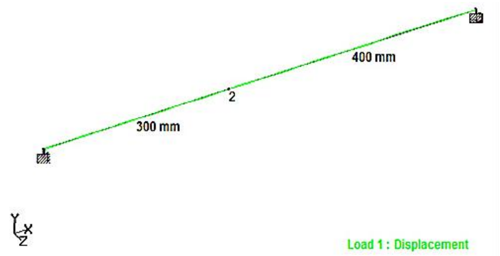

Fig 3.4: Nodal Displacement

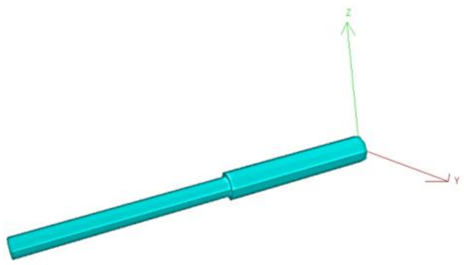

Fig 3.5: 3D view of the beam section

Table 3.1 Section Properties of materials at Case 1

Pro	Section (mm)	Area (mm^2)	Ixx (mm^4) X 10^5	Iyy (mm^4) X 10^5	J (mm^4) X 10^5	Materials
1	Cir 60	2400	4.584	4.584	9.168	Al
2	Cir 40	1200	1.147	1.147	2.295	Steel

Table 3.2 Reaction at node of beam at Case 1

Node	Horizontal	Vertical	Horizontal	Moment		
	F_x(kN)	F_y(kN)	F_z(kN)	M_x	M_y	M_z
1	-294.535	0	0	0	0	0
3	-105.465	0	0	0	0	0

Table 3.3 Nodal Solution of beam at Case 1

Node	X (mm)	Y (mm)	Z (mm)	Resultant (mm)	Rx Rad	Ry Rad	Rz Rad
1	0	0	0	0	0	0	0
2	0.271	0	0	0.271	0	0	0
3	0	0	0	0	0	0	0

CASE 2:
Analysis of support reaction and nodal displacement of stepped beam at one end is fixed and another is applying axial load 5kn.
Where,
$A_1 = 600\, mm^2,\ A_2 = 70\ \ mm^2$
$l_1 = 100\, mm, l_2 = 100mm$

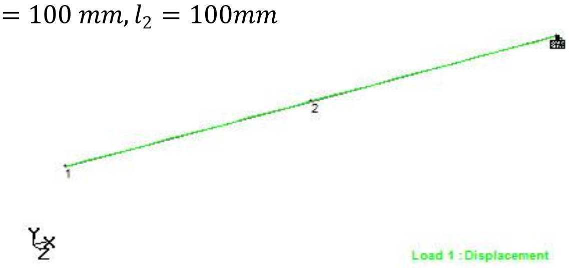

Fig 3.6: Nodal Displacement of Beam

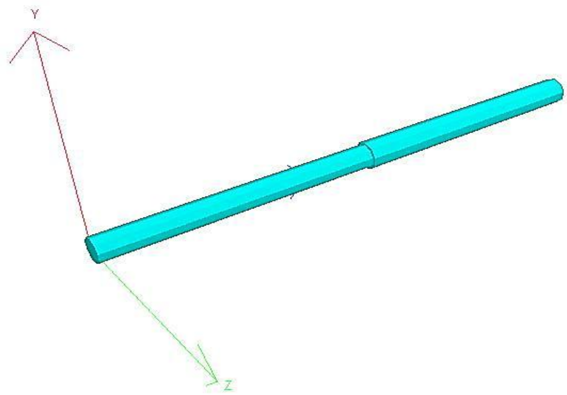

Fig 3.7: 3D view of the beam section

Table 3.4 Section Properties of materials at Case 2

Pro	Section (mm)	Area (mm^2)	Ixx (mm^4) X 10^5	Iyy (mm^4) X 10^5	J (mm^4) X 10^5	Materials
1	Cir 10	100	0.079	0.079	0.159	Al
2	Cir 10	70	0.039	0.039	0.078	Steel

Table 3.5 Reaction at node of beam at Case 2

Node	Horizontal	Vertical	Horizontal	Moment		
	F_x(kN)	F_y(kN)	F_z(kN)	M_x	M_y	M_z
3	-5	0	0	0	0	0

Table 3.6 Nodal Solution of beam at Case 2

Node	X (mm)	Y (mm)	Z (mm)	Resultant (mm)	Rx Rad	Ry Rad	Rz Rad
1	0.090	0	0	0.090	0	0	0
2	0.073	0	0	0.073	0	0	0
3	0	0	0	0	0	0	0

CASE: 3
Analysis of support reaction, nodal displacement and stress of a stepped beam at both end are fixed and axial load is 15 kN at node 2
Where,
$A_1 = 600\, mm^2,\ A_2 = 300\ \ mm^2$
$l_1 = 300\, mm, l_2 = 400mm$

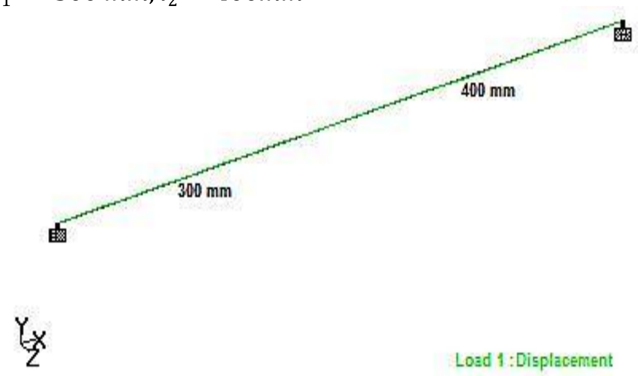

Fig 3.8: Nodal Displacement

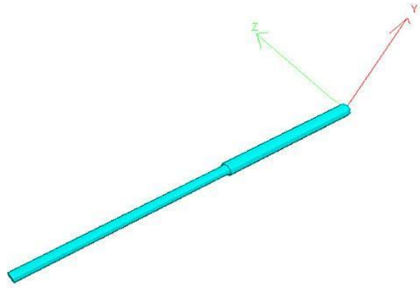

Fig 3.9: 3D view of the beam section

Table 3.7 Section Properties of materials at Case 3

Pro	Section (mm)	Area (mm^2)	Ixx (mm^4) X 10^5	Iyy (mm^4) X 10^5	J (mm^4) X 10^5	Materials
1	Cir 30	600	2.865	2.865	5.730	Al
2	Cir 20	300	0.716	0.716	4.431	br

Table 3.8 Reaction at node of beam at Case 3

Node	Horizontal	Vertical	Horizontal	Moment		
	F_x(kN)	F_y(kN)	F_z(kN)	M_x	M_y	M_z
1	-12.955	0	0	0	0	0
3	-2.045	0	0	0	0	0

Table 3.9 Nodal Solution of beam at Case 3

Node	X (mm)	Y (mm)	Z (mm)	Resultant (mm)	Rx Rad	Ry Rad	Rz Rad
1	0	0	0	0	0	0	0
2	0.040	0	0	0.040	0	0	0
3	0	0	0	0	0	0	0

IV. RESULTS AND DISCUSSIONS

The various results obtained when considering different cases and solve by using finite element method and STAAD.Pro software and we get the result are shown in Table 4.1.

CASE	FORCE	FEM	STAAD.PRO
1	Reaction	R_1= 305.9 kN R_3= 94.041 kN	R_1= 294.535 kN R_3= 105.465 kN
	Nodal Displacement	δ_2= 0. 36593 mm	δ_2= 0. 271 mm
	Stress	S_1= 50.96 N/mm^2 S_2= 39.18 N/mm^2	S_1= 57.834 N/mm^2 S_2= 43.942 N/mm^2
2	Reaction	R_1= 5 kN	R_1= 5 kN
	Nodal Displacement	δ_2= 0.07143 mm δ_3= 0. 1071 mm	δ_2= 0. 073 mm δ_2= 0. 090 mm
	Stress	S_1= 65.28 N/mm^2 S_2= 46.58 N/mm^2	S_1=71.439 N/mm^2 S_2= 50.034 N/mm^2
3	Reaction	R_1= 10.38 kN R_3= 4.618 kN	R_1= 12.955 kN R_3= 2.045 kN
	Nodal Displacement	δ_2= 0. 0742 mm	δ_2= 0. 04 mm
	Stress	S_1= 16.8 N/mm^2 S_2= 15.4 N/mm^2	S_1= 19.09 N/mm^2 S_2= 10.345 N/mm^2

Table 4.1: Various results obtained from FEM and STAAD.Pro

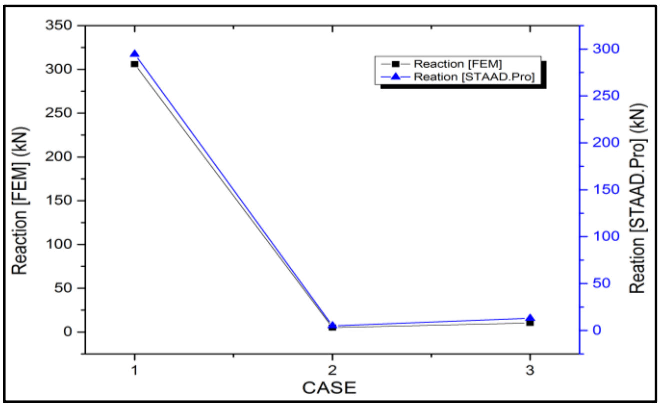

Fig 4.1: Graph plotted b/w Reaction obtained from FEM and STAAD.Pro different cases

The above fig 4.1 has been indicate that the reactions are obtained from considering different cases and solves using finite element method and STAAD.Pro software and plotted the graph between these results and compared the solution result. In this fig 4.1 indicating that the both results are approximate same.

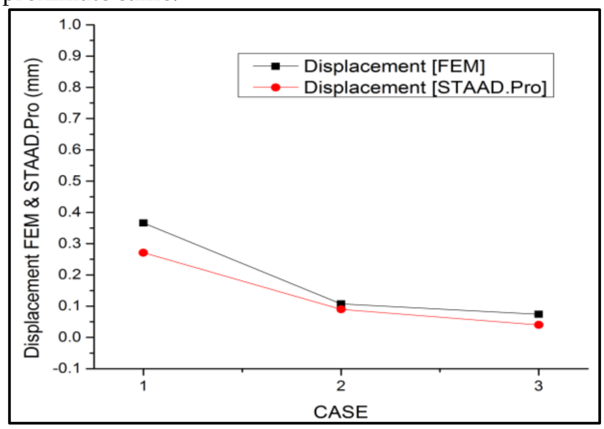

Fig 4.2: Graph plotted b/w displacement obtained from FEM and STAAD.Pro at different cases

The above fig 4.2 has been shows that the displacements are obtained from considering different cases and solves using finite element method and STAAD.Pro software and plotted the graph between these results and compared the solution result.

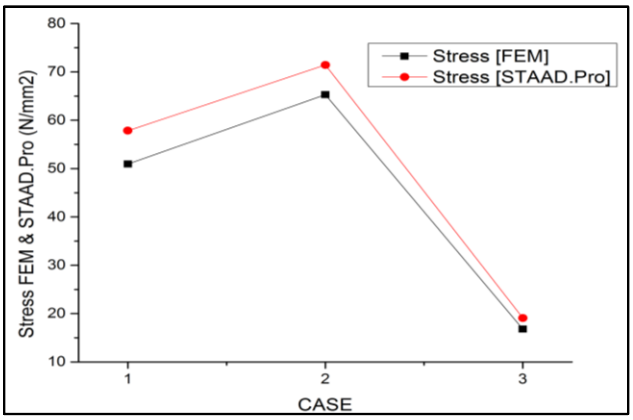

Fig 4.3: Graph plotted b/w Reaction obtained from FEM and

STAAD.Pro different cases

The above fig 4.3 has been indicate that the stress are obtained from considering different cases and solves using finite element method and STAAD.Pro software and plotted the graph between these results and compared the solution result.

In all above fig 4.1, 4.2 and 4.3 indicating that the both results are approximate same.

V. CONCLUSION

- The Results exhibit hardening type nonlinearity. Stiffness of the system increases with deflection. It shows the effect of stretching of mid-plane of the stepped beam.
- From the discussions regarding the position of the stiffener, it can be safely said that the maximum stiffness or the lowest deflection can be obtained when the stiffener is placed at the center of the beam.
- Change of the stiffener geometry (while maintaining the cross-sectional area constant) apparently doesn't have significant effect on the stiffness of beam.
- From result and discussion it is concluded that the when solving the problem using finite element method, it is complicated and time taken but using STAAD.Pro software it is obtained the approximate same result. So it is helpful for solving the problem in less time and it is also developed the software skill.

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