Abstract: For coordination of over current relay linear programming techniques such as simplex method, dual simplex method and two phase simplex method are used. Another way is to apply artificial intelligent based method. This paper presents particle swarm optimization method to find minimum time dial setting (TDS) for coordination of over current relay. Then this method is compared with linear programming method. The algorithm has been implemented in matlab for radial and parallel feeder system.

Index Terms: Constrained optimization, Optimal relay coordination, Particle swarm optimization (PSO), linear programming (LP), Time dial setting (TDS), Optimization problem.

I. INTRODUCTION
The most common type of faults are shunt faults which results in sudden increase in magnitude of current therefore over current protection is most common type of protection[1]. The protective relay must be able to discriminate between normal operating conditions and fault conditions. The protective relay should operate as fast as possible for its zone faults which is known as primary protection. In case when primary protection fails to operate than back up protection should come in to action after sufficient coordination time interval. The property of selectivity must not be violate in the operation therefore primary and back up protection must be coordinate properly.[3][7] Effective coordination of relay can be carried can be done with both time and magnitude of current. The fault clearing time should be optimum so it causes minimum damage to system and also minimum interruption of load at remote consumers. [12] Therefore relay coordination can be considered as optimization problem which is affected by constraint to ensure minimum time of operation under specified selectivity constraints. Therefore it is constraints optimization problem.[2] In this paper relay coordination problem is presented as linear programming (LP) problem i.e pick up setting of relays fixed and operating time of relay is in linear proportion with time multiplier setting. This paper present how particle swarm optimization can be implemented to optimally coordinate relays. Two general cases have been taken under consideration. The basic difference between the formulations of the problem is that if plug setting multiplier is kept constant or unchanged or fixed than in that case the coordination problem is consider as linear programming problem(LPP). If plug setting multiplier is of continuous values than the problem is consider as non linear programming(NLP) and if plug setting multiplier is in distinct valued steps than the problem is consider as mixed integer non linear programming (MINLP)[2][10] For easy evaluation and simplicity of one of the most vital problem of power system protection field it is evaluated as linear programming problem (LPP). Linear programming technique is simple way to deal with complicated non linear problems in the field of optimization. This makes the complicated concept of relay coordination merely easy and simple.[8]

II. PROBLEM FORMULATION
A. Problem statement
The main purpose of this problem is to obtain minimum operation time of relays so that there could be minimum damage to system. For that the problem can be formulated using either linear programming or nonlinear programming technique. To avoid complexity in solving the problem we have considered linear formulation of problem where plug setting (PSM) of relay is fixed and time of operation is in linear proportion with time multiplier setting (TMS).

In general in coordination problem of over current relay main intention is to minimize the algebraic sum of operation time of total relays.

Objective Function:-

\[ S = \min \sum_{i=1}^{n} t_i \]

B. Relay Characteristics
Relay characteristics of over current relay depends on the type of relay. The general characteristics formula for operating time of over current relay is given by

\[ Top = \frac{a}{(PSM)^b - 1} \cdot TDS + C \]

Where PSM = plug setting multiplier, it is the ratio of current at short circuit to pick up current of relay.
A, B and C are constants shown in the table [4]
TDS is time dial setting of relay, having distinct step values in a range from 0 to 1 in step of 0.1.

<table>
<thead>
<tr>
<th>S.N.</th>
<th>Characteristic</th>
<th>Standard</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>A</td>
</tr>
<tr>
<td>1</td>
<td>Short time</td>
<td>AREVA</td>
<td>0.05</td>
</tr>
<tr>
<td>2</td>
<td>Standard</td>
<td>IEC</td>
<td>0.14</td>
</tr>
<tr>
<td>3</td>
<td>Very inverse</td>
<td>IEC</td>
<td>13.5</td>
</tr>
<tr>
<td>4</td>
<td>Extremely</td>
<td>IEC</td>
<td>80</td>
</tr>
</tbody>
</table>
C. Coordination constraints
The relays in the network must be coordinated by criterion

\[ t_{2k} - t_{1k} \geq MCT \]

Where,

- \( t_{2k} \) = operating time of back up relay
- \( MCT \) = Minimum coordination time

In this paper the minimum coordination time is taken as 0.25.[9]

D. Boundary limits on the relay operating time
Relays in the network should operate fast at the same time there should be sufficient minimum time between primary and back up relay.

\[ t_{1k \min} \leq t_{1k} \leq t_{1k \max} \]

E. Relay coordination characteristics

Fig. 1 Relay characteristics
In this paper normal IDMT relay is considered

III. METHODS
In this paper two methods are used as shown below:

1. Linear Programming Technique
It is most simple and popular technique used for optimization problem. A Linear Programming method seeks to maximize or minimize a linear function, subject to a set of linear constraints. This paper present simplex method.[11]

The linear model consists of the following components:
- A set of decision variables, an objective function, a set of constraints which is expressed as
- Inequality constraints: \((A) \mathbf{X} \leq \mathbf{b}\)
- Equality constraints: \((A_{eq}) \mathbf{X} = \mathbf{beq}\)
- Bounded optimization constraints:
  - Lower bound \( \leq \mathbf{X} \leq \) Upperbound

The structure of Linear programming technique involves following:
- Focused parameter: The required result is the time multiplier setting (TMS).

(b) Target parameter: objective function is to minimize the time of operation.
(c) Constraints: Inequality boundary and equation that represents optimization problem.
(d) Restricted variables: decision variable involved is not negative i.e. time multiplier can not be negative.

2. Particle swarm optimization
PSO is a population based optimization method which is inspired by the sociological behavior associated with bird flocking and fish schooling [5]. The main feature of PSO is that no gradient information derived from objective function. The main idea of the PSO algorithm is to maintain a population of particles which is referred as “swarm” in which each particle represents a potential solution to the objective function. Each particle in swarm can memorize its current position which can be determined by evolution of objective function, velocity and best visited position during its flying tour in the problem search space which is its personal best solution (Pbest). The personal best solution represents highest fitness value for the particle in minimization task higher fitness means the position having a smaller function value. The best position among all pbest is known as global best position. The particles of swarm are assumed to travel the problem search space in a discrete time steps. At each iteration the velocity of each particle is changed with current velocity and distance from pbest and gbest as

\[
v_{i}^{k+1} = v_{i}^{k} + c_{1} \cdot \text{rand}() \cdot \frac{(pbest_{i} - s_{i}^{k})}{\Delta t} + c_{2} \cdot \text{rand}() \cdot \frac{(gbest_{i} - s_{i}^{k})}{\Delta t}
\]

Where
- \( v_{i}^{k} = \) ith velocity component at iteration \( k \)
- \( \text{rand}() = \) random number between 0 and 1
- \( s_{i}^{k} = \) current position in the ith dimension
- \( c_{1}, c_{2} = \) acceleration coefficients
- \( pbest_{i} = \) personal best position in the ith dimension
- \( gbest_{i} = \) global best position in the ith dimension
- \( \Delta t = \) time step

Fig. 2 Three components of the velocity update equation.
Normally the range of velocity is kept between -Vmax to Vmax so that the particle might not fly out of the search space. Now the search space is limited by -Xmax to Xmax so that value of Vmax is set so that Vmax = kXmax, where 0.1 ≤ k ≤ 1 [5]. After that each particle is free to update its position by its current velocity to explore the problem search space to get better results as follows

\[ s_i^{k+1} = s_i^k + v_i^{k+1} \Delta t \]

Attempting to increase the rate of convergence of the standard PSO algorithm to global optimum, the inertia weight has been introduced in the velocity update equation [6]. The inertia weight is scaling factor associated with the velocity during the previous time step. According to this modification proposed, (1) is modified to

\[ v_i^{k+1} = w v_i^k + c_1 \cdot \text{rand()} \cdot \frac{(pbest_i - s_i^k)}{\Delta t} + c_2 \cdot \text{rand()} \cdot \frac{(gbest_i - s_i^k)}{\Delta t} \]

\[ ..............(2) \]

Where w is the inertia weight.

The inertia weight governs how much of the previous velocity should be retained from previous time step. In this work linearly decreasing inertia weight is used [5]. The inertia weight is set to decrease linearly from 0.9 to 0.4 during the course of a simulation. This setting allows the particles to explore a large area at the start of the simulation (when the inertia weight is large), and to refine the search later by using a small inertia weight. In addition, damping the oscillations of the particles around gbest is another advantage gained by using a decreasing inertia weight. These oscillations are recorded when a large constant inertia weight is used. Accordingly, damping such oscillations assists the particles of the swarm to converge to the global optimal solution. [5]

It is important to realize that the velocity term models the rate of change in the position of the particle. Therefore, the changes induced by the velocity update equation represent acceleration, which explains the name of acceleration coefficients for the constant sand. The acceleration coefficients can be thought of as a balance between Exploration (searching for a good solution) and exploitation (taking advantage of someone else’s success).

Too little exploration and the particles will all converge on the first good solution encountered, while too little exploitation and the particles will never converge, i.e., they will just keep searching. There is another way of looking at this rather than behaviors (exploration and exploitation). What must be properly balanced is individuality and sociality, i.e., traits that influence behavior. Ideally, individuals prefer being individualistic yet they still like to know what others have achieved so that they can learn from.

IV. RESULTS

A. Radial System

The two section simple radial feeder is shown in Fig. 3. It is clearly seen that if fault occurs beyond bus B (Zone k), the relay RB at bus B, operates firstly as a primary protection. The relay RA at bus A, serves as a back-up protection and operates after certain time delay.

![Fig. 3 Radial system](image)

If the time of operation of relay RB, is set at 0.2s, the relay RA is coordinated in such a way that it should operate at time 0.2+MCT. The objective function for the study is to determine optimal operating time. Time of operation is in linear proportion with TMS. Therefore the objective function can be written as:

\[ S = \min \sum_{i=1}^{n} K_i t_i \]

\[ ..............(3) \]

Where, \[ K_i = \frac{0.14}{(PSM)^{0.02} - 1} \]

For this approach, a two bus radial feeder system was considered in Fig. 3. The maximum fault current just beyond bus A and bus Bare 4000A and 3000A respectively, the initial plug setting of relays are taken as 100% .the CT ratios are 300: 1 and 100: 1 at bus A and bus B respectively. PSM is calculated on the basis of maximum fault current and CT ratios available. TMS for relay RB is set at 0.1 and the TMS for relay RA is to be found out using the mentioned Linear Programming and Particle swarm optimization respectively. Minimum time of operation for both relay is taken as 0.2s and MCT is taken as 0.25s for this case study. Calculation for ki is done using equation specified and is as shown in Table II.

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Fault location</th>
<th>Relay-1</th>
<th>Relay-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Just beyond Bus A</td>
<td>2.63</td>
<td>------</td>
</tr>
<tr>
<td>2</td>
<td>Just beyond B</td>
<td>2.97</td>
<td>2</td>
</tr>
</tbody>
</table>

-------- indicates that fault is not seen by relay

These lead to the following optimization equations:

Objective function:

\[ s = 2.63x_1 + 2x_2 \]

\[ ..............(4) \]

Subject to coordination constraint,

\[ 2.97x_1 - 2x_2 \geq 0.25 \]

\[ ..............(5) \]

And lower bounds on the relay is decided by minimum operating time of relay.(considering 0.2s)
The problem is of two dimensional (d = 2). The optimization equations were solved using Particle swarm optimization in the MATLAB environment. The population size was taken as 50 and the number of iterations was 200.

For simplicity we have considered the plug setting of each relay to be 1 and corresponding CT ratios are 300:1. Relay 2 provides backup protection to relay 3 in case if fault occurs on upper line. Similarly relay 1 will provide back up to relay 4 in case if fault occurs on lower line connected between bus A and bus B. We have considered for our studies that fault occurred on first line at the half of length of line and then in second case fault occurred on lower bus at middle of the line.

Total fault current in each case is taken to be 4000A. Minimum Coordination Time for this exercise is considered as 0.5s.

TABLE V: Calculation of Constant for relay (for system-2)

<table>
<thead>
<tr>
<th>Fault point</th>
<th>Relay</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Relay current</td>
<td>10</td>
<td>3.33</td>
<td>3.33</td>
<td>......</td>
</tr>
<tr>
<td>K</td>
<td>2.97</td>
<td>5.749</td>
<td>5.749</td>
<td>......</td>
<td>3.33</td>
</tr>
<tr>
<td>F2</td>
<td>Relay current</td>
<td>3.33</td>
<td>10</td>
<td>......</td>
<td>3.33</td>
</tr>
<tr>
<td>K</td>
<td>5.749</td>
<td>2.97</td>
<td>......</td>
<td>5.749</td>
<td></td>
</tr>
</tbody>
</table>

... Indicates that faults are not in the tripping direction

Objective function:

\[ S = 2.97x_1 + 2.97x_2 + 5.749x_3 + 5.749x_4 \]

Coordinate constraints,

\[
\begin{align*}
5.749x_2 & - 5.749x_3 \geq 0.5 \\
5.749x_3 & - 5.749x_1 \geq 0.5
\end{align*}
\]

And the minimum time constraints for the said optimization problem:

\[
\begin{align*}
2.970x_1 & \geq 0.2 \\
2.970x_2 & \geq 0.2 \\
5.749x_3 & \geq 0.2 \\
5.749x_4 & \geq 0.2
\end{align*}
\]

In this problem there are four relays incorporated so it is a four dimensional problem. The population size taken as 50 and iteration count is 200. The algorithm is a kind of swarm intelligence and after every iteration count the result is updated to a new value. The TDS obtained are shown in table VI and the results are obtained and compared with linear programming technique. The total time of operation by both the methods is tabulated in table VII.

TABLE VI: Total optimal time of operation using PSO and LP (for system-1)

<table>
<thead>
<tr>
<th>Total Operating Time</th>
<th>Linear Programming</th>
<th>PSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>0.6047</td>
<td>0.5985</td>
</tr>
</tbody>
</table>

B. Parallel Feeder System

Fig. 4 Parallel feeder system

In above case, a simple parallel feeder system is shown. In the figure only 4 breakers are shown incorporated with 4 relays from which two connected with breaker 1 and 2 is non-directional relay whereas relays 3 and 4 are directional relays whose direction is towards source from the bus.

For simplicity we have considered the plug setting of each relay to be 1 and corresponding CT ratios are 300:1. Relay 2 provides backup protection to relay 3 in case if fault occurs on upper line. Similarly relay 1 will provide back up to relay 4 in case if fault occurs on lower line connected between bus A and bus B. We have considered for our studies that fault occurred on first line at the half of length of line and then in second case fault occurred on lower bus at middle of the line.

Total fault current in each case is taken to be 4000A. Minimum Coordination Time for this exercise is considered as 0.5s.

TABLE VII: Total optimal time using LP and PSO

<table>
<thead>
<tr>
<th>Total operating time (sec)</th>
<th>Linear programming</th>
<th>Particle swarm optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>1.1455</td>
<td>1.127</td>
</tr>
</tbody>
</table>
V. CONCLUSION

The time of operation for the relay coordination is found out using both the techniques. The result obtained using these techniques are certainly far better than conventional one, but the result obtained in both the cases using particle swarm optimization is better than Linear Programming Technique. In case of digital relays fractional values of time dial setting are also kept and hence the optimal time of operation is achieved, which improves system performance and reliability. The results obtained can be compared with other techniques. There are several constraints handling methods with which one can go for further result comparison with other optimization techniques as well as with the same technique. Comparatively good solution is available to optimization problem.

REFERENCES
