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DYNAMICS OF SLENDER STRUCTURAL MEMBER HAVING CRACKS

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Abstract: Structural discontinuity present in the structural member can potentially threaten the safe performance of the component and components may also fail during the service which may be dangerous to life. The main focus of the thesis is to develop the method to detect the location and the size of the crack in slender structural members using changes in natural frequency of transverse vibration. The cracks considered, are external crack which are normal to the axis and has straight front. Also, the approach for the detection of the crack location and the crack size for the multi-span beam have been developed. Effect of the crack location and the crack size on the natural frequency has been studied and presented for various end conditions for multi-span beam. The same approach is verified to demonstrate the effectiveness of the crack location. The whole study indicates that the detection of crack condition i.e. crack location and size is possible based on changes in the natural frequency of the transverse vibration.

Keywords: Crack detection, natural frequency, vibration based inspection, Non-Destructive Testing (NDT), Transverse Vibration, multiple crack detection, crack detection in multi-span beam, effect of crack location and size on natural frequency.

I. INTRODUCTION

The increasing demands of higher productivity and economical design lead to higher operating speeds of machinery and efficient use of materials through light weight structures. These trends make occurrence of resonant conditions more frequent during the operation of machinery and reduce the reliability of the system. It is required that structures must safely work during its service life. But, damages initiate a breakdown period on the structures. Cracks are among the most encountered damage types in the structures. Beam type structures are commonly used in steel construction and machinery industry. So, the structural safety of beam is very important due to its practical importance. Structural discontinuity can potentially threaten their safe performance and components may also fail during the service which may be dangerous to life. It is desired to monitor the component and detect the damage or discontinuities at earliest possible stage. A wide group of Non- destructive testing (NDT) techniques used in science and industry to detect these discontinuities by evaluating the properties of a material, component or system without causing damage. Common NDT methods include X-ray technique, ultra-sonic technique, magnetic-particle technique, liquid penetrant method, radiographic method, remote visual inspection (RVI), eddy-current testing, and low coherence interferometry. But these all methods are time consuming, laborious and expensive for long components like pipes, rails and slender beams. This has motivated for development of alternative methods. One of the potential candidate as new approach for the damage detection is Vibration Based Inspection (VBI). Vibration Based Inspection (VBI) makes use of either the local or global effects produced by the crack. The changes in natural frequency form a basis for the global effect. This is useful for the component having limited or full access. Limited access is generally the case for the pipes located in the hazardous area of nuclear plants or under-sea pipelines.

II. LITERATURE SURVEY

A. Free Vibration of Beams

Extensive study has been done on the free vibration of the components like beams, shafts, pipes, truss or frames etc. with or without cracks with different geometry like non-uniform cross-section and various uniform cross sections and with different boundary conditions like pined-pined, fixed-free, fixed-fixed, free-free conditions. Modeling is based on Euler-Bernoulli or Timoshenko beam theory (G. D. Gounaris et al. [1]; Viola et al. [2]; Lin [3]). Euler-Bernoulli theory is suited for slender beams and Timoshenko theory is suited for short beams where rotational inertia and shear deformation are not negligible

B. Crack Modeling

Modeling of crack is critical due to:

- Physical discontinuity at the crack location,
- Large stress concentration at crack tip due to square-root singularity,
- Crack results in significant localization of the deformation which in turn increase flexibility or decrease stiffness or change in damping

Ostachowicz and Krawczuk [4] have presented method of analysis to study the effect of double-sided and single-sided open cracks upon the frequencies of the natural flexural vibrations in a cantilever beam. They concluded that the effect of the double-sided crack is smaller than the effect of the single sided cracks with same crack depth and position.

C. Crack detection Techniques for beam using Vibration based approach

Suitable techniques used for crack detection using vibration based approach are:

- Frequency Based Damage Detection (FBDD)
- Mode-Shape Based Damage Detection (MBDD)
- Wavelet Based Damage Detection (WBDD)

D. Approaches for Multiple Cracks

Case of multiple cracks is more complex than the case of the single crack. Component with multiple cracks can be analyzed by two approaches:

Forward Approach

- In this approach, attempt is made to find out modal parameters knowing the crack location and the size.

Inverse Approach

- In this approach, attempt is made to find out crack location and size knowing the modal parameters

III. AIM AND SCOPE

Keeping above details in the consideration, investigation has been carried out for the following objectives:

- To study the suitability of the rotational spring method for modelling transverse vibration of slender structural member with crack
- To develop a method for the solution of inverse problem involving single and multiple cracks in slender structural members.
- To study the effect of the crack location and the crack size on the natural frequency of the free transverse vibration of the slender structural members for various end conditions.
- To apply the developed approach to the slender structural members with different end conditions i.e. fixed-free (Cantilever beam), pined-pined (Simply Supported beam), fixed-fixed etc.
- To develop method for the solution of inverse problem involving single crack in straight pipes.
- To apply rotational spring approach to detect the double-sided open-crack present in the structural member.
- To study free vibration of the multi-span beam having crack and apply rotational spring approach to detect the crack location and the crack size.
- To study effect of the crack location and the crack size on the natural frequency of the free vibration of the multi-span beam for various end .
- To carry out experiments to validate the accuracy of the theoretical prediction.

IV. IDETECTION OF SINGLE CRACK IN STRUCTURAL MEMBERS

Crack in the beam has been modelled by the rotational spring. Crack detection approach has been developed to detect the crack in the beam with fixed-free (cantilever) and pinned-pinned (simply supported), fixed- fixed end conditions. Also, the effect of the crack location and the crack size on the natural frequency has been studied for the various end conditions like, fixed-free (cantilever), pinned-pinned (simply supported) and fixed- fixed end conditions.

A. Analysis of beam with crack using Rotational Spring Approach

Consider the free vibration of the beam with a single crack. Applying the rotational spring method for representing the crack, spring stiffness must be defined.

Dimarogonas and Paipetis [5] defined the bending spring constant `$K_t$' in vicinity of the cracked section of a beam with orthogonal cross-section of width `b' and height `h'. When lateral crack of uniform depth `a' exists, from crack energy function

$$K_t = \frac{1}{C}$$

$$C = \frac{5{:}346h\ f(a/h)}{EI}$$

$f(a/h) = 1.8624(a/h)^2 – 3.95(a/h)^3 + 16.375(a/h)^4 - 37.226(a/h)^5 + 76.81(a/h)^6 - 126.9(a/h)^7 + 172(a/h)^8 - 143.97(a/h)^9 + 66.56(a/h)^{10}$

B. Detection of crack location and the crack size

The experimental natural frequencies of the different cantilever beam can be utilized in the mathematical formulation presented here, to predict the crack location and the crack size. Corresponding to first three natural frequencies, the variation of normalized crack depth (a/h) with normalized crack location (x/L) is obtained, as shown in the Figure 1. The intersection of these three graphs will precisely predict the crack location and the crack size.

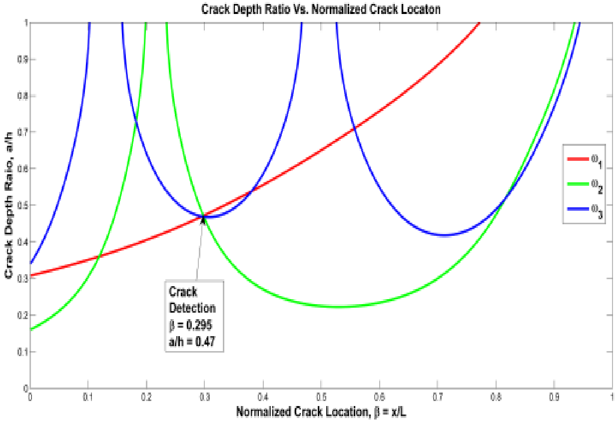

Figure 1: Detection of the Crack in Cantilever Beam having a Single Crack

C. Effect of Normalized crack location and crack depth ratio on natural frequency

The natural frequency is greatly affected by the crack depth and the crack location. To illustrate this, the condition |A| = 0 is incorporated for different crack depth ratios, crack locations and frequencies.

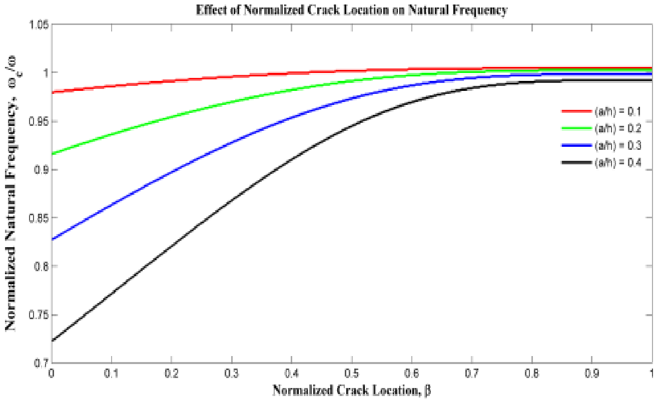

Figure 2: Effect of Normalized Crack Location on the Normalized Natural Frequency for a Beam with Fixed-Free (Cantilever) end conditions

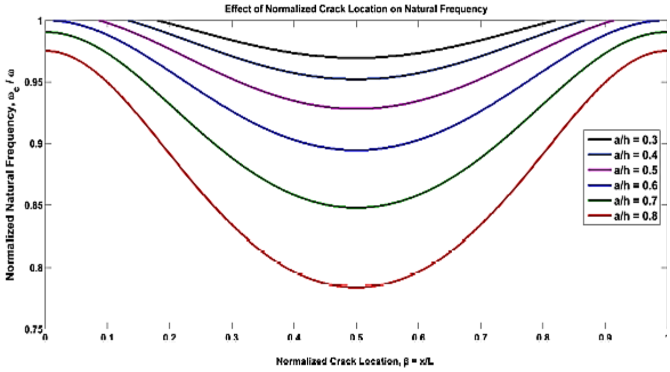

Figure 3: Effect of Normalized Crack Location on the Normalized Natural Frequency for a Beam with Pinned-Pinned (Simply Supported) end conditions

V. DETECTION OF SINGLE CRACK IN

A. Straight Pipes

The straight pipe with single crack is shown in the Figure 4. The same method above mentioned for the crack detection in the beam has been employed to detect the crack on the pipe. First three natural frequencies have been found and those are used to plot variation of the rotational spring stiffness K with normalized crack location. Intersection of these three plots corresponding to first three natural frequencies gives the location of the crack and the spring stiffness of the crack. The validation of this approach has been given later in this chapter in section Experimental Validation.

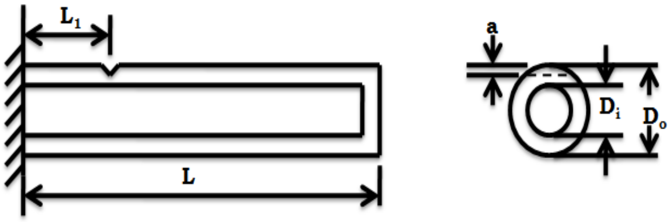

Figure 4: Straight Pipe with a Single Transverse Open-Crack

B. Double Sided opens Crack

The double-sided open crack is illustrated in the Figure5. In practice, this kind of cracks occurs in case of two sided bending of the beam.

Using the f(a/h) function for the crack defined by Hellan [6] and Haisty and Springer [7], the rotational spring stiffness at the crack location can be defined as,

$$K = \frac{bh^2E}{9\pi} f\left(\frac{a}{h}\right)$$

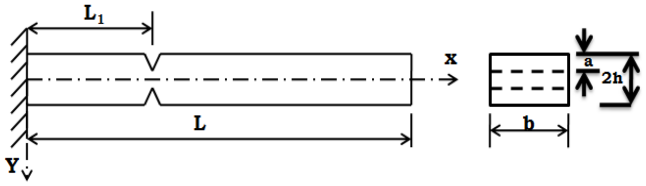

Figure 5: Cantilever Beam with a Double-sided Open Crack

Where, f(a/h)= 0.5335(a/h)2 – 0.929(a/h)3 + 3.5(a/h)4- 3.181(a/h)5 + 5.793(a/h)6

VI. EXPERIMENTAL VALIDATION

The experimental setup is developed for obtaining natural frequencies of cracked and uncracked beam (Refer Figure 3.11). The beam dimensions and its material properties are as follows: Length (L) = 1 m, width (b) = 0.04 m and depth (h) = 0.01 m, mass density (ρ) = 7860 kg/m3 and modulus of elasticity (E) = 210 GPa.

Figure 6: Experimental Set-up for Crack Detection of a Cantilever Beam with a Single Crack

The cracks are developed using wire-cut electro-discharge machining and natural frequencies are measured using LMS make FFT analyzer shown in Figure 7. FFT Analyzer is connected to the computer and using LMS Test Express Software provided by LMS, FFT plot of the acceleration of the beam can be obtained as shown in the Figure 8. Peaks in this plot show the corresponding natural frequencies.

Figure 7: LMS Make FFT Analyzer

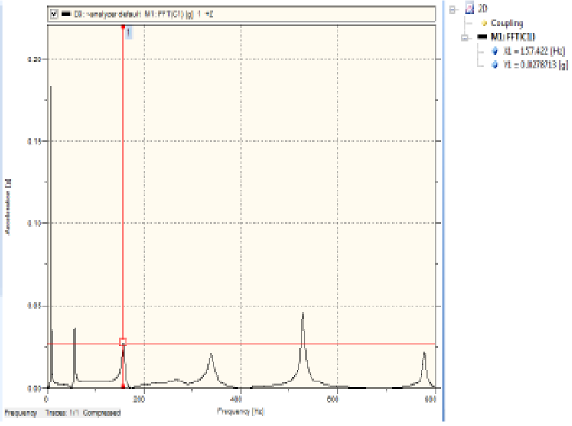

Figure 8: FFT plot obtained to find the Natural Frequencies of Free Transverse Vibration of the Beam

Case of Beam with Fixed-Free (Cantilever) end conditions

Normalized Crack Location, (β= x/L)	Crack Depth Ratio, (a/h)	Natural Frequency, ώ₁ (Hz)	Natural Frequency, ώ₂ (Hz)	Natural Frequency, ώ₃ (Hz)	Predicted Crack Location, ($β_P$) (Presented Method)	Error in Predicted Crack Location, (%)	Predicted Crack Depth, $(a/h)_P$ (Presented Method)	Error in Predicted Crack Depth, (%)
Uncracked		8.3493	52.308	146.39	-	-	-	-
0.1	0.5	8.0681	51.682	146.06	0.099	1	0.486	2.8
0.2	0.5	8.16	52.303	145.46	0.206	3	0.4785	4.3
0.3	0.5	8.22	52.091	143.79	0.295	1.67	0.47	6
0.4	0.5	8.2721	51.548	145.03	0.402	-0.5	0.4736	5.28
0.5	0.5	8.3088	51.224	146.41	0.504	-0.8	0.45	10

Table 1: Comparison of Actual and Predicted Crack Location and Crack Size by Natural Frequency for a Beam with Fixed-Free (Cantilever) end conditions.

As can be seen from the Table 1, the maximum error between the actual and predicted crack location is 3%.

Case of Beam with Pinned-Pinned (Simply Supported) end conditions

Normalized Crack Location, (β = x/L)	Crack Depth Ratio, (a/h)	Natural Frequency, ώ₁ (Hz)	Natural Frequency, ώ₂ (Hz)	Natural Frequency, ώ₃ (Hz)	Predicted Location, ($β_P$) (Presented Method)	Error in Predicted Crack Location, (%)	Predicted Crack Depth, $(a/h)_P$ (Presented Method)	Error in Predicted Crack Depth, (%)
Uncracked		23.43	93.621	210.27	-	-	-	-
0.1	0.5	23.376	92.847	207.04	0.102	-2	0.478	4.4
0.2	0.5	23.243	91.724	206.3	0.202	-1	0.4762	4.76
0.3	0.5	23.085	91.8	209.87	0.302	-0.67	0.4835	3.3
0.4	0.5	22.947	92.907	208.59	0.401	-0.25	0.4811	3.78
0.5	0.5	22.938	93.621	146.41	0.504	-0.8	0.4672	6.56

Table 2: Comparison of Actual and Predicted Crack Location and Crack Size by Natural frequency for a Beam with Pinned-Pinned (Simply Supported) end conditions

As can be seen from the Table 2, the maximum error between the actual and predicted crack location is 2%. The maximum error between the actual and predicted crack size is 6:56%.

Case of Beam with Fixed-Fixed end conditions

Normalized Crack Location, (β= x/L)	Crack Depth Ratio, (a/h)	Natural Frequency, ώ₁ (Hz)	Natural Frequency, ώ₂ (Hz)	Natural Frequency, ώ₃ (Hz)	Predicted Location, ($β_P$) (Presented Method)	Error in Predicted Crack Location, (%)	Predicted Crack Depth, $(a/h)_P$ (Presented Method)	Error in Predicted Crack Depth, (%)
Uncracked		53.114	146.34	286.68	-	-	-	-
0.1	0.5	52.457	146.02	286.66	0.101	-1	0.4895	2.1
0.2	0.5	53.093	145.35	281.72	0.193	3.5	0.4966	0.68
0.3	0.5	52.942	143.54	284.76	0.303	-1	0.4831	3.38
0.4	0.5	52.489	144.87	285.34	0.401	-0.25	0.4811	3.78
0.5	0.5	52.241	146.33	280.4	0.5	0	0.484	3.2

Table 3: Comparison of Actual and Predicted Crack Location and Crack Size by Natural Frequency for a Beam with Fixed-Fixed end conditions

As can be seen from the Table 3, the maximum error between the actual and predicted crack location is 3:5%. The maximum error between the actual and predicted crack size is 3:78%.

Firstly, free vibration of the beam has been studied in this section. Then, the crack detection approach representing crack as rotational spring has been developed and the same has been applied for various end conditions, i.e. fixed-free, pinned-pinned and fixed- fixed end conditions. Also, the effect of the crack location and the crack depth for the fixed-free, pinned-pinned and fixed- fixed end condition has been examined. Crack parameters are predicted for various end conditions and the predicted crack parameters are quite accurate with maximum error in crack location to be 3:5% and in the crack depth to be 10%.

VII. CRACK DETECTION IN MULTI-SPAN BEAM

The experimental setup is developed for obtaining natural frequencies of cracked and uncracked beam. The beam dimensions and its material properties are as follows: Length (L) = 1.8 m, width (b) = 0.04 m and depth (h) = 0.01 m, hinged support at 0.9 m, mass density (ρ) = 7860 kg/m3 and modulus of elasticity (E) = 210 GPa.

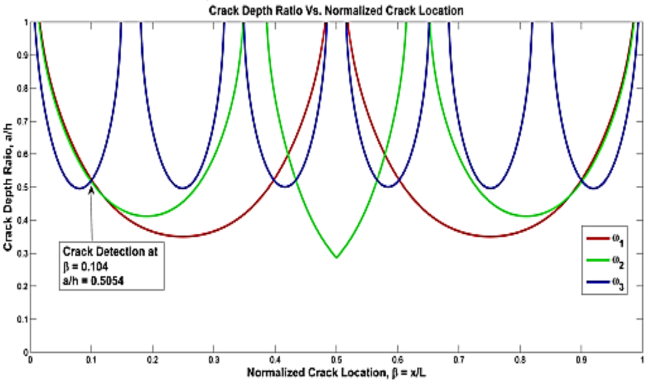

Figure 9: Crack Detection in the Pinned-Pinned Beam with Hinged Support at center

Corresponding to first three natural frequencies, the variation of normalized crack depth (a/h) with normalized location (x/L) is obtained, as shown in the Figure 9. The intersection of these three curves will precisely predict the crack location and the size. The Figure 9 is prepared for the natural frequencies corresponding to β= 0.1 and a/h = 0.5. The predicted location and the crack size are β= 0.104 and a/h = 0.5054, respectively.

Normalized Crack Location, (β= x/L)	Crack Depth Ratio, (a/h)	Natural Frequency, ώ₁ (Hz)	Natural Frequency, ώ₂ (Hz)	Natural Frequency, ώ₃ (Hz)	Predicted Crack Location, ($β_P$) (Presented Method)	Error in Predicted Crack Location, (%)	Predicted Crack Depth Ratio, $(a/h)_P$ (Presented Method)	Error in Dicted Crack Depth Ratio, (%)
Uncracked		28.931	45.185	260.01	-	-	-	-
0.1	0.5	28.762	44.843	256.31	0.1010	-1	0.5172	-3.44
0.2	0.5	28.519	44.607	258.63	0.2005	-0.25	0.5075	-1.5
0.25	0.5	28.451	44.704	255.69	0.252	-0.8	0.524	-4.8
0.3	0.5	28.528	44.979	258.73	0.3007	-0.23	0.514	-2.8
0.4	0.5	28.768	45.112	256.19	0.3985	0.375	0.5205	-4.1

Table 4: Comparison of Actual and Predicted Crack Location and Crack Size by Natural frequency for Pinned-Pinned Beam with Hinged Support at center

As can be seen from the Table 4, the maximum error between the actual and predicted crack location is 1%. The maximum error between the actual and predicted crack size is 4:8%.

VIII. CONCLUSION

Using this approach, damage detection can be done using natural frequency. The followings are the conclusions made from the present study:

- The presented method to detect crack location and size is fast and efficient.
- Crack with larger crack depth ratio (a/h) imparts greater reductions in natural frequency than that of the smaller crack depth ratio. Hence, the accuracy of results improves as rack depth increases.
- Crack present near to fixed end imparts greater reductions in natural frequency than that to present at away from the fixed end.
- Presented approach also gives good results for detecting crack on the multi-span beam.
- This method can also be applied to detect crack in the pipes filled with and without containing pressurized fluid.

In future this approach can be extended to detect multiple cracks on the multi-span beam. Approach further more can be extended for detection of the crack on the frame-types structures.

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