International Journal For Technological Research In Engineering
Volume 5, Issue 5, January-2018 *ISSN (Online): 2347 - 4718*

EXPERIMENTAL AND NUMERICAL INVESTIGATIONS OF THE EFFECT OF BENDING LOAD ON CURVED BEAM

Harinath S.P[1], Srivathsa B.S[2]
[1,2]Department of Mechanical Engineering, Nitte Meenakshi Institute of Technology, Bengaluru, India

Abstract: Curved beams represent an important class of machine members which find their application in components such as crane hook, c – clamp, frames of presses etc. The stress analysis of the critical section of the curved beam is a crucial step in its design. There are two analytical methods used for stress analysis of curved beams: a plane elasticity formulation and Winkler's theory. The Winkler's theory has long been the primary means of curved beam stress analysis in engineering practice. The present work is measuring inner and outer strain of the curved beam with the help of strain gauges and it also describes the method of strain analysis of a U – shaped specimen, the base of which represents a curved beam using the standard Winkler's theory and a follow on experimental strain analysis using strain gauges. The specimen is loaded such that a known bending moment is applied to it. The circumferential stresses along the critical section of the curved beam are determined using Winkler's theory. During the follow on experimental procedure, a mild steel U – shaped specimen is instrumented with strain gauges along the critical section. The gauges are used to measure the circumferential strains along the critical section. The circumferential stresses are then calculated using Hooke's law. Together, numerical method and lab experiment illustrates many essential elements of experimental stress analysis of a curved beam.
Keywords: Curved beam, Winkler's theory, strain gauges, Hooke's law

I. INTRODUCTION

Historically beams were squared timbers but are also metal, stone, or combinations of wood and metal such as a flitch beam. Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (e.g., loads due to an earthquake or wind or in tension to resist rafter thrust as a tie beam or (usually) compression as a collar beam). The loads carried by a beam are transferred to columns, walls, or girders, which then transfer the force to adjacent structural compression members. In light frame construction joists may rest on beams. A beam is a structural element that is capable of withstanding load primarily by resisting against bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment. Beams are characterized by their profile (shape of cross-section), their length, and their material. Beams are traditionally descriptions of building or civil engineering structural elements, but smaller structures such as truck or automobile frames, machine frames, and other mechanical or structural systems contain beam structures that are designed and analyzed in a similar fashion. Curved beams are the parts of machine members found in C clamps, crane hooks, frames machines, planers etc. In straight beams the neutral axis of the section coincides with its centroidal axis and the stress distribution in the beam is linear. But in the case of curved beams the neutral axis of is shifted towards the center of curvature of the beam causing a non-linear [hyperbolic] distribution of stress. The neutral axis lies between the centroidal axis and the center of curvature and will always be present within the curved beams.

In other words beam in which the neutral axis in the unloaded condition is curved instead of straight or if the beam is originally curved before applying the bending moment, are termed as —"Curved Beams"

The beam theory can also be applied to curved beams allowing the stress to be determined for shapes including crane hooks and rings. When the dimensions of the cross section are small compared to the radius of curvature of the longitudinal axis the bending theory can be relatively accurate. The present work objectives are mentioned below.

II. OBJECTIVE

- To study importance of curved beams in machine elements
- To formulate Winkler's theory for calculating inner and outer strain
- To obtain the inner and outer strain experimentally by utilizing strain gauges
- A comparison study is carried out between numerical and experimental results

III. WINKLER-BATCH THEORY

Consider a curved beam subjected to bending moment M as shown in the figure. The distribution of stress in curved flexural member is determined by using the following assumptions:

i) The material of the beam is perfectly homogeneous [i.e., same material throughout] and isotropic [i.e., equal elastic properties in all directions]
ii) The cross section has an axis of symmetry in a plane along the length of the beam.
iii) The material of the beam obeys Hooke's law.
iv) The transverse sections which are plane before bending remain plane after bending also.
v) Each layer of the beam is free to expand or contract, independent of the layer above or below it.
vi) The Young's modulus is same both in tension and compression

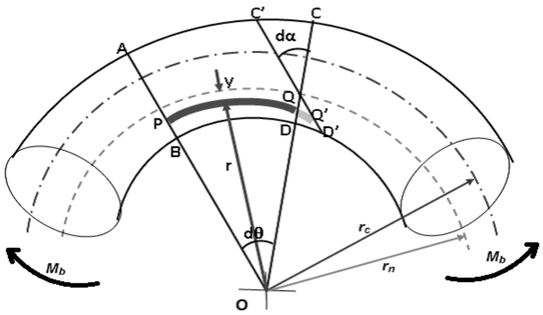

Fig 1 shows the curve beam specifications

Table 1: Symbols specifications

ε = strain e = eccentricity (r_c - r_n) (m) c_c = Distance from centroid axis to inner surface. (m) c_i = Distance from neutral axis to inner surface. (m) c_o = Distance from neutral axis to outer surface. (m) dφ= Surface rotation resulting from bending stress σ = stress (N/m^2)	E = Young's Modulus = σ /e (N/m^2) y = distance of surface from neutral surface (m). r_n = Radius of neutral axis (m). r_c = Radius of centroid (m). r = Radius of axis under consideration (m). I = Moment of Inertia (m^4 - more normally cm^4) Z = section modulus = I/y_{max}(m^3 - more normally cm^3)

$$\varepsilon = (r - r_n)\frac{d\emptyset}{r\emptyset} \quad \text{Eqn 1}$$

The strain is clearly 0 when r = at the neutral axis and is maximum when r = the outer radius of the beam ($r = r_o$) Using the relationship of stress/strain = E the normal stress is simply,

$$\sigma = E\varepsilon = E(r - r_n)\frac{d\emptyset}{r\emptyset} \text{Eqn 2}$$

The location of the neutral axis is obtained from summing the product of the normal stress and the area elements over the whole area and equating to 0

$$\int_A \sigma\, dA = \frac{E\, d\emptyset}{\emptyset}\int_A \frac{r - r_n}{r}\, dA = 0 \quad \text{Eqn. 3}$$

Reduce to,

$$A - r_n \int_A \frac{dA}{r} = 0 \qquad \text{Eqn. 4}$$

Therefore,

$$r_n = \frac{A}{\int_A \frac{dA}{r}} \text{Eqn. 5}$$

Neutral Axis for a Rectangular Section,

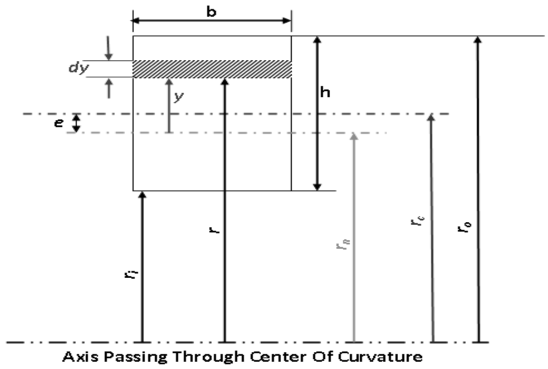

Fig 2 shows the specification of the beam element

$$r_n = \frac{A}{\int_A \frac{dA}{r}} = \frac{b(r_o - r_i)}{\int_{r_i}^{r_o} b(\frac{dr}{r})} = \frac{r_o - r_i}{ln\frac{r_o}{r_i}} \text{Eqn. 6}$$

Eccentricity = e = (r_c – r_n) Eqn.7

IV. CURVED BEAM IN BENDING

The stress resulting from an applied bending moment is derived from the fact that the resisting moment is simple the integral over the whole section of the moment arm from the neutral axis (y) multiplied by σ dA = dF. Moment equilibrium is achieved if,

$$M = \int_A (r - r_n)\sigma\, dA = \frac{E\, d\emptyset}{\emptyset}\int_A \frac{(r - r_n)^2 dA}{r} \text{Eqn. 8}$$

$$= \frac{E\, d\emptyset}{\emptyset}\int_A \frac{(r^2) - 2rr_n + r_n{}^2)\, dA}{r} \text{Eqn. 9}$$

The centroid of a section,

$$r_c = \frac{1}{A}\int_A r.dA\, \text{Eqn. 10}$$

Therefore,

$$M = E\frac{d\emptyset}{\emptyset} Ae \quad \text{Eqn. 11}$$

$$\sigma = \frac{M.(r - r_n)}{Aer} \text{Eqn. 12}$$

The maximum stress occurs at either the inner or outer strain,

$$\sigma_i = \frac{M.c_i}{Aer_i}, \quad \sigma_o = \frac{M.c_o}{Aer_o}$$

The curved beam flexure formula is in reasonable agreement for beams with a ratio of curvature to beam depth of r_c/h of > 5 (rectangular section). As the beam curvature/depth radius increases the difference between the maximum stress calculated by curved beam formula and the normal beam formula reduces.

V. CURVED BEAM LOADING

A U-shaped specimen which is made up of mild-steel will be used for the following experiment to find the inner and the outer strain.

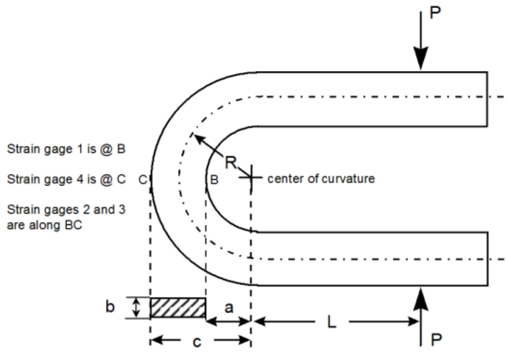

Fig 3 shows the curved beam specimen

Where,

- P = load applied in N
- b = width in mm
- c = distance from center of curvature to outer radius in mm
- a= distance from center of curvature to inner radius in mm
- L= distance of loading from center of curvature in mm
- R= radius of the neutral axis in mm

VI. RESULTS

Table 2 shows the calculated and obtained result

Load in Kg	Inner strain Theoretical	Inner strain Practical	Out strain Theoretical	Out strain Practical	Error in %
5	0.036	0.033	0.065	0.059	8.33
10	0.095	0.068	0.132	0.120	8.54
15	0.116	0.100	0.189	0.174	8.94
20	0.127	0.103	0.199	0.183	9.33
25	0.149	0.132	0.251	0.232	9.56

VII. CONCLUSION

- The importance of curved beams has been studied by considering their applications
- Formulating Winkler's theory for calculating inner and outer strain was accomplished
- The inner and outer strain experimentally by utilizing strain gauges were obtained for different loads
- A comparative study was carried out between numerical and experimental results by achieving an error of less than 10%

REFERENCES

[1] M.F .SPotts, T.E. Shoup "Design of Machine Elements", pearson Edn, 2006
[2] Joseph E. Shigley, Charles R. Mischke and Thomas H. Brown "Standard Handbook of Machine Design"
[3] TORE DAHLBERG "Procedure to calculate deflections of curved beams" - Int. J. Engng Ed. Vol. 20, No. 3, pp. 503-513, 2004
[4] Aimin Yu "Theory and Application of Naturally Curved and Twisted Beams with Closed Thin-Walled Cross Sections" - Journal of Mechanical Engineering 55(2009)12, 733-741
[5] Gaydon, F.A. (1968).The rectangle under general equilibrium loading in generalized plane stress.Proc. R. Soc. A 283,356
[6] Sarma, P.V.B.A.S., Ramachandra Rao,B.S., and Gopalacharyulu, S. (1975). Eigenfunction Solution for plane stress problems of curved beams. Int.J.Engng.Sci., 13.149-59.
[7] Dym,C.L., and Williams, H.L(2011)., "Stress and Displacement estimates for Arches," Journal of structural engineering,137(1),pp.xxx-xxx.
[8] ERIK PERSSON (2012) "Stress and strength analysis of curved glulam beams with box cross section" Masters Dissertation LUND University.

