OPTIMIZATION OF CANTILEVER FIN USING DIFFERENT TYPES OF CROSS-SECTIONS

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Abstract: Fins are crucial components wherever heat transfer is necessary. If combustion or heat generation process takes place inside any closed chamber, it is necessary to remove the heat and this is where fins come into consideration. They are used to transfer heat from walls of the chamber to surrounding air. Heat transfer takes place in fins through conduction and convection in practical case. The transfer of heat hugely depends on geometrical conditions of fins, environmental conditions and material properties of fins. It is important to have proper material and dimensions of fins so that maximum heat transfer can take place. This study focuses on the optimum dimensions of a fin (with adiabatic tips) with different cross-sections for maximum heat transfer to take place and the deformation taking place due to increase in length, provided that volume remains the same in all cases.
Keywords: Heat Transfer, Fins Optimization, Fins Dimensions, Fins Deformation, ANSYS, MATLAB

I. INTRODUCTION

In any type of closed chamber, if the surface available is insufficient to transfer the necessary heat with respect to the available temperature drop and coefficient of convective heat transfer according to the external conditions, extended projections or surfaces are used. These projections are called fins [1]. Fins help to dissipate extra heat in the environment. If fins are not used where required, it may result into high stresses in material due to temperature drop and this may result into failure of the component.

The heat transfers from the inner walls of the vessel or chamber where heat generation takes place by conduction to fins. Conduction and convection takes place simultaneously through fins and heat is transferred to the ambient air. Fins can be classified into different types. However, in this study, fins that were considered were those with adiabatic tips. The different cross-sections considered were square, rectangle, circle and triangle. Deformation taking place in fin due to its length and own weight were calculated and values were noted.

Heat transfer by fins with adiabatic tips can be given by [2]:

$$Q = \sqrt{hPkA}\theta_b \tanh mL \quad .....\text{Eq. 1}$$

Where,
h = convective heat transfer coefficient
k = conductivity of the material
θ_B= temperature difference
m = $\sqrt{\frac{hP}{kA}}$

For all the cross-sections considered, values of $k\ and\ \theta_B$ will be constant. Differentiating Q with respect to t and equating it to 0, optimum fin thickness and optimum fin length can be obtained for maximum heat transfer. Length was calculated using the value of thickness obtained as

Length = Volume/Area

With the available dimensions, deformations in fins were evaluated using the formula [3]:

$$\delta = \frac{Wl^3}{8EI}$$

Where,
W = weight of the fin itself (m*g)
E = modulus of elasticity of the material
I = moment of inertia of the shape
In all the cases below, values of E and ρ (density) were

$$E = 6.89*10^{10}\ N/m^2$$
$$\rho = 2700\ N/m^3$$

(All the values of the dimension were taken in meters. However, for visualization purpose, the values in only graph were converted to mm).

II. FINS WITH RECTANGULAR CROSS-SECTIONAL AREA

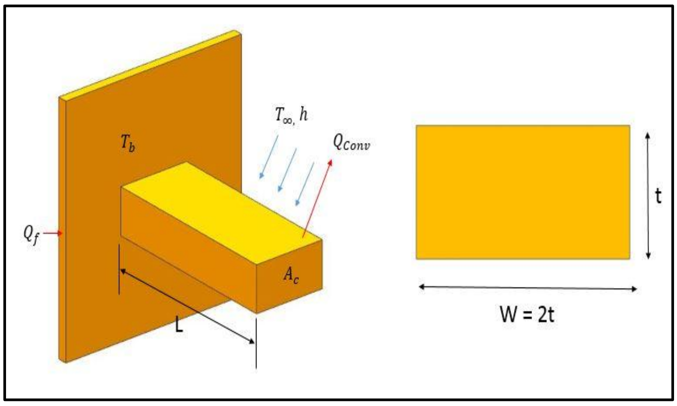

Figure 1: 3D Model of Rectangular Cantilever fin

In rectangular fins, the thickness of one fin was assumed to be t (in meters) and the width was taken as 2t. Hence, area of the cross section was $2t^2$. As mentioned earlier, volume of each fin was assumed as 1 m^3.

After putting the values of area and perimeter, the equation of Q obtained was

$$Q = a\, t^{\frac{3}{2}} \tanh \quad b\, t^{-\left(\frac{5}{2}\right)} \text{....Eq. 2}$$

Where, $a = 2\theta_b\sqrt{3hk}$

$$b = V\sqrt{\frac{3h}{4k}}$$

To obtain optimum thickness,

$$\frac{dQ}{dt} = 0$$

Solving the above differentiation, we achieved

$$\sin 2\alpha = \frac{10\alpha}{3} \qquad \text{.....Eq. 3}$$

Where, $\alpha = bt^{-\left(\frac{5}{2}\right)}$

The value of α was obtained by solving Eq. 3 in MATLAB. The value of α obtained was 0.92.

Hence, by values of α, b and V, different values of thickness, width and length of the fin were be obtained with given values of ratio of convective heat coefficient and heat conductivity of material. Deformation due to own weight of the fin was also obtained.

Table 1: Length And Thickness Comaparison in Rectangular Fin

h/k	T (m)	W (m)	L (m)	δ (m)
1	0.976	1.952	0.524	6.01E-09
0.5	0.849	1.699	0.692	3.17E-08
0.1	0.615	1.231	1.318	1.51E-06
0.05	0.536	1.072	1.739	7.97E06
0.01	0.388	0.777	3.311	0.00037
0.005	0.338	0.676	4.369	0.00200
0.001	0.245	0.490	8.317	0.09534
0.0005	0.213	0.426	10.974	0.50322

$[\alpha = 0.92;\ V = 1m^3]$

The variation of change in length and thickness can be visualized in the following graph:

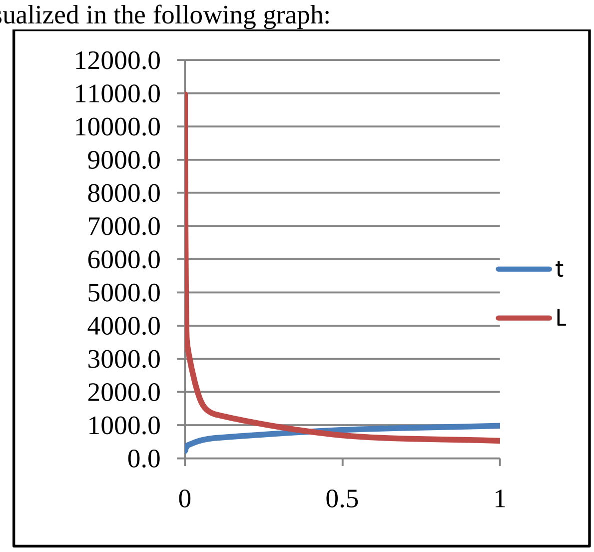

Figure 2 Change of Length and Thickness in Rectangular Fin

III. FINS WITH SQUARE CROSS-SECTION:

In fins with square cross-section, the dimensions (in meters) assumed were:

Thickness = t

Width = t

Hence, cross-sectional area = t^2 m^2; perimeter = 4t m.

In this case, the equation of heat transfer obtained was

$$Q = a\, t^{\frac{3}{2}} \tanh \quad b\, t^{-\left(\frac{5}{2}\right)} \qquad \text{....Eq. 4}$$

Where,

$$a = 2\theta_b \sqrt{hk}$$

$$b = \sqrt{\frac{4h}{k}}\, V$$

Taking,

$$\frac{dQ}{dt} = 0$$

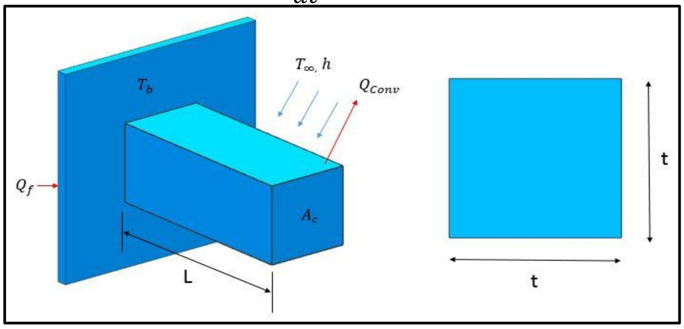

Figure 3: 3D Model of Square Cantilever fin

The value of α derived was same as that in case of rectangular cross-section, i.e. 0.92

Where, $\alpha = bt^{-\left(\frac{5}{2}\right)}$

From given values of (h/k) and V, value of b was obtained and thus, thickness was calculated by using the relation of α with b and t. The values which were derived, along with deformation in fin are listed below in the table:

Table 2**:** Length And Thickness Comparison in Squre Fin

h/k	T (m)	W (m)	L (m)	δ (m)
1	1.364	1.364	0.537	1.39E-08
0.5	1.187	1.186	0.708	7.31E-08
0.1	0.860	0.860	1.349	3.48E-06
0.05	0.749	0.749	1.780	1.84E-05
0.01	0.543	0.543	3.390	0.00087
0.005	0.472	0.472	4.473	0.00015
0.001	0.342	0.342	8.515	0.00552
0.0005	0.298	0.298	11.236	1.15923

The graph of value of thickness and length w.r.t. different values of (h/k) obtained in this case of fin was:

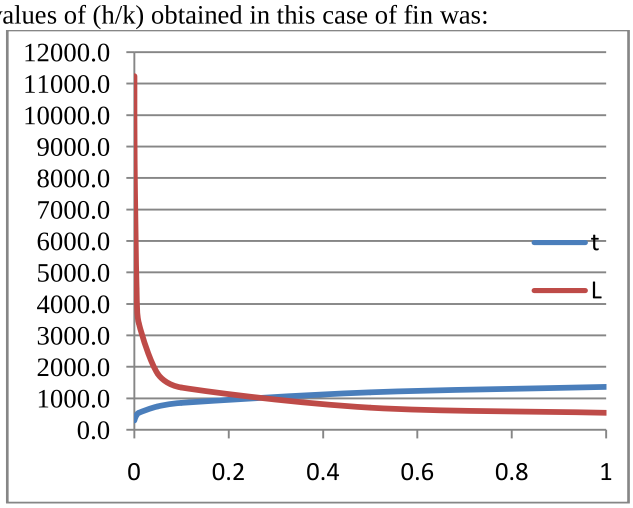

Figure 4: Change of Length and Thickness in Squre Fin

IV. FINS WITH TRIANGULAR CROSS-SECTION

In triangular cross-sectional fin, in this study, it is assumed that the triangle is equilateral. Dimensions that were taken for this fin:
Length of each side of triangle: t
Cross-sectional area: $(\sqrt{3}/4)*t^2$.
This case will see no width as a triangle has only base and altitude.
If the fin has triangular cross-section, heat transfer taking place through it can be given by:

$$Q = a\, t^{\frac{3}{2}} \tanh \quad b\, t^{-\left(\frac{5}{2}\right)} \text{....Eq. 5}$$

Where,

$$a = 3^{\frac{3}{4}}\theta_b \sqrt{\frac{hk}{4}}$$

$$b = \frac{8}{\sqrt[4]{3}} \sqrt{\frac{h}{k}}\, V$$

It is necessary to differentiate this equation w.r.t thickness and equating it to 0 in order to obtain optimum thickness (in this case: length of each side of triangle) such that heat transfer taking place is maximum.
Like in above case, the solution to Eq. 5 achieved was

$$\sin 2\alpha = \frac{10\alpha}{3}$$

And value of α was calculated as 0.92.
Hence, different values of't' and 'L' calculated for maximum heat transfer in triangular fin were as below. Deformation taking place due to length of fin and its own weight was also calculated.

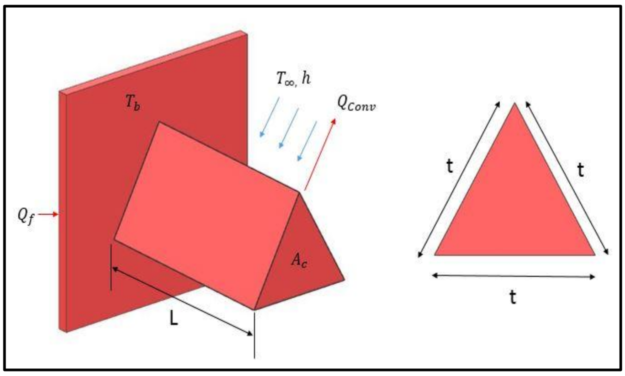

Figure 5: 3D Model of Triangular Cantilever fin

Table 3: Length And Thickness Comparison in Tringular Fin

h/k	t (m)	L (m)	δ (m)
1	2.128	0.509	8.77E-09
0.5	1.852	0.672	4.63E-08
0.1	1.342	1.280	2.20E-06
0.05	1.168	1.690	1.16E-05
0.01	0.847	3.217	0.00055
0.005	0.737	4.245	0.00292
0.001	0.534	8.081	0.13896
0.0005	0.465	10.663	0.73343

The graph of 'L' and't' obtained in this case was:

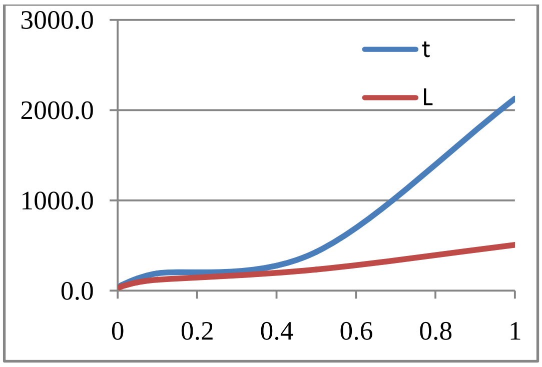

Figure 6: Change of Length and Thickness in Tringular Fin

V. FINS WITH CIRCULAR CROSS-SECTIONAL AREA

In circular fin, the diameter was the dimension on which other parameters depended. Assuming its value to be't', following was concluded:
Cross-sectional area: $(\pi*t^4)/4$
Circumference: $\pi*t$

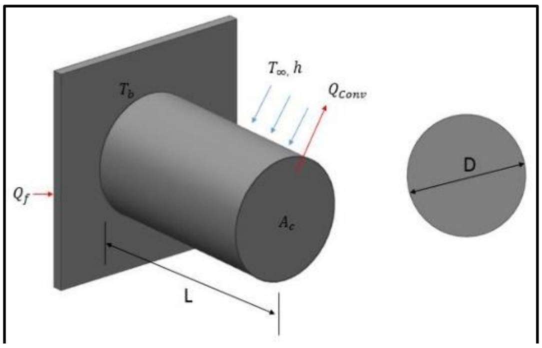

Figure 7: 3D Model of Circular Cantilever Fin

Heat transfer in fin with circular cross-section is written as that in other cases as all of them are of same type, i.e. with adiabatic tip. Hence,

$$Q = a\, t^{\frac{3}{2}} \tanh \quad b\, t^{-\left(\frac{5}{2}\right)}$$

Here,

$$a = \frac{\pi}{2}\theta_b\sqrt{hk}$$

$$b = \sqrt{\frac{h}{k}\frac{8V}{\pi}}$$

Hence, solution for which heat transfer is maximum is obtained as $\alpha = 0.92$.
The table below illustrates different optimum values of't' and 'L' for different values of (h/k) and also the deformation taking place due to its own weight and corresponding length:

Table 4: Length And Thickness Comaprison in Circular Fins

h/k	t (in m)	L (in m)	δ (m)
1	1.502	0.563	1.94E-08
0.5	1.308	0.744	1.02E-07
0.1	0.948	1.416	4.87E-06
0.05	0.825	1.868	2.57E-05
0.01	0.598	3.557	0.00122
0.005	0.520	4.694	0.00646
0.001	0.377	8.936	0.30734
0.0005	0.328	11.792	1.622155

The graph for fin with circular cross-section plotted was:

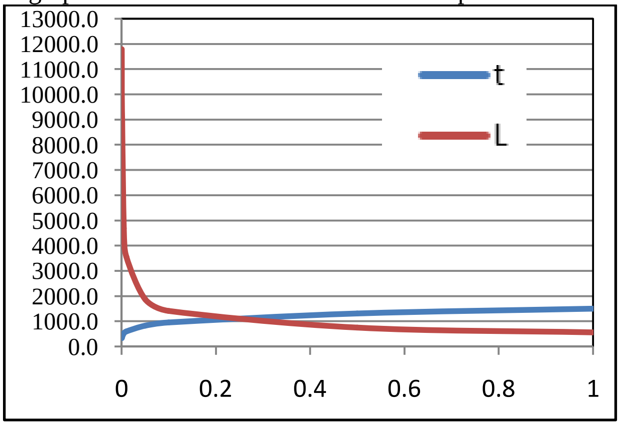

Figure 8: Change of Length and Thickness in Circular Fins

VI. CONCLUSION

From the study, it was concluded that for a given value an h/k, the deformation in fin with rectangular cross-section was minimum among all the four cross-sections. Hence, for a particular value of h/k, a fin with rectangular cross-section will induce minimum stress. Moreover, the dimensions will be such that for that particular cross-section and value of h/k, heat transfer taking place will be maximum.

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