

DESIGN AND IMPLEMENTATION OF PERFECT RECONSTRUCTED FILTER BANK AND COMPARED TWO APPROACHES

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Abstract: The paper describes the design and implementation of perfect reconstruction filter bank for advanced communication satellite. The theory of filter bank satisfying the perfect reconstruction using proto FIR filter of length $N = 2 \cdot M \cdot m$; where $M = \text{No. of channel}$. The paper formulates the design approach of any arbitrary length proto FIR filter for perfect reconstruction cosine modulated filter bank and FARROW structure based filter bank. Further a design for M -channel transmultiplexers for GEO communication satellite is described through mathematical modeling. A new set of objective functions has been derived to satisfy the transmultiplexers requirements based on these approaches. This work is done under the technology development program of ISRO. In this paper a comparison of the DCT and FARROW approaches are describing.

Keywords: perfect reconstruction, filter bank, lattice structure, SRC, TMUX

I. INTRODUCTION

Transmultiplexer(TMUX), a class of DSP algorithms used to construct multirate digital filter banks. Traditionally TMUX were deployed for the FDM-TDM-FDM translation for on board processing in satellite communication. TMUX is well suited for simultaneous transmission of many data signals through a common channel. TMUX performed by mainly two parts analysis part and synthesis part. At analysis side first input signal is dividing into many signals that signals pass through a decimator. Decimator provides a decimation of the input signal and then that decimated signal is passing through a parallel structure of prototype filter. The channelization is performed by that filter. At synthesis side channelized signal passed through the interpolator provides the interpolation of the channelized signal. That signal pass through again parallel structure of prototype filter and combine the all signal give one composite output. Perfect reconstruction (PR) property is required for restoration of the original signals in difficult application of TMUX. Perfect reconstruction is achieved by various filter banks to use of Para unitary property of filter. Any system is called perfect reconstruction when there is not have any phase distortion, not have amplitude distortion and free from aliasing effect. In this paper multicarrier modulation-demodulation can be accomplished efficiently by a digital TMUX using a discrete cosine transform(DCT) and FARROW based approach also compare the both approaches than conclude that DCT is better than FARROW based approach.

A. FARROW BASED TMUX:

Farrow structure based multimode TMUX consists of variable frequency shifter and Farrow base variable integer sampling rate conversion (SRC). Each integer SRC block is design using in a fixed set of sub filters. There is only need to modify fractional delay values to perform any one set of integer SRC. Farrow structure designed that poly phase component of M -channel interpolation and decimation filter, approximates Nyquist filter. Using this method Nyquist filter with small arbitrarily error and different edges of pass band archive [10]. Therefore, TMUX is archiving PR via proper designed of sub filters. In this session low pass filter use in synthesis and analysis filter bank. The advantage of the Farrow based TMUX is the elimination of the low pass filter (using fixed set of sub filter), which also results in a different way to design the sub filters of the Farrow structure, with constraints in the whole frequency range $[-\pi, \pi]$.

B. DCT BASED TMUX:

DCT approach dividing mainly two-part lattice structure and cosine modulated matrix. First is Lattice structure work as a filter, set the value of filter and second is poly phase component of matrix cosine modulated filter banks(CMFB) is lossless, losslessness ensured that analysis-synthesis system satisfied PR. Analysis-synthesis system obtained by cosine modulation, an efficient implementation is derived using $2M$ poly phase component of prototype filter and DCT matrix. The whole processes not have any phase distortion, amplitude distortion and aliasing [2].

C. PAPER NOTATION:

1. The both approaches satisfy perfect reconstruction.
2. Filter banks design for an arbitrary number of M channel.
3. Filter length $N = 2 \cdot m \cdot M$
Where $M = \text{Number of channel}$
 $m = \text{arbitrary constant}$
4. The analysis and synthesis filters are of equal length (N).

D. PAPER OUTLINES:

Following this introduction, Section II describes the FARROW structure, Section III describes the DCT structure, and Section IV gives a comparison of all TMUX approaches. Section V concludes the paper.

II. FARROW BASED TMUX

Farrow structure in such a way that the overall filter is a linear combination of linear-phase finite-impulse response

(FIR) sub filters $H(z)$ weighted with u^k [5]. When u is fixed, the overall filter approximates a low-pass filter with the fractional delay; provided that the sub filters have been designed in a proper manner then PR is archived.

The major advantage of using the Farrow structure is that when a new fractional delay is desired, the sub filters do not have to be redesigned but it suffers to adjust the value of u .

Fig 1 shows that, the Farrow structure is composed of sub filters $C_k(z)$ which is fixed linear-phase finite-length impulse response (FIR), where $k = 0, 1, \dots, L$ with either a symmetric ($k = \text{even}$) or antisymmetric ($k = \text{odd}$) impulse response each input sample and its corresponding output sample [6].

The transfer function of the Farrow structure is given by,

$$H(z) = \sum_{k=0}^L C_k(z)u^k, |u| \leq 0.5 \quad (1)$$

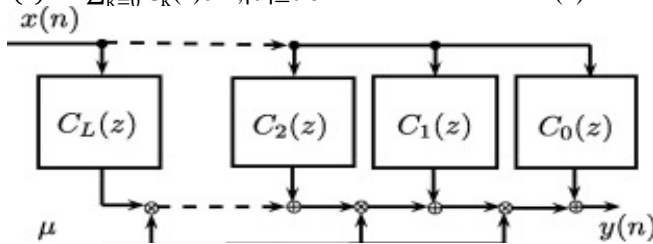


Figure 1 Farrow structure

where μ denotes fractional delay value and the sub filters $C_k(z)$ denotes linear-phase FIR filters of either odd or even order, say N_1 , and with symmetric impulse responses $c_k(n)$ for even, with antisymmetric impulse responses $c_k(n)$ for odd i.e., $c_k(n) = c_k(N_1 - n)$ for even and $c_k(n) = -c_k(N_1 - n)$ for odd [7]. Consider that we have, T_{in} is the sampling period at the input $x(n)$ and T_{out} is the sampling period at the output $y(n)$, respectively, then for [8]

Even Order sub filters: $[n_{in} + u(n_{in})]T_{in} = n_{out} T_{out}$
 Odd Order sub filters: $[n_{in} + 0.5 + u(n_{in})]T_{in} = n_{out} T_{out}$

Where n_{in} = input sample index and (n_{out}) = output sample index. The Farrow structure generates a delayed (with a delay of μ) version of the input signal if μ is constant for all input samples. However, the Farrow structure can perform SRC if μ changes for every input sample. The fractional delay (μ) value defines the time difference between its input samples and corresponds to its output sample [8].

The Farrow structure can realize the poly phase components of a general low-pass filter which means that different integer SRC ratios can be used to implement interpolators/decimators with fixed set of sub filters.

o Fig 2 shows that, TMUX consists of up sampling and down sampling by R_p ; low-pass interpolation/decimation filters, i.e., $G_p(z)$ for interpolation and $\hat{G}_p(z)$ for decimation; and adjustable frequency shifters, i.e., frequency shifts by ω_p and $\hat{\omega}_p$. Assuming the T_p is the sampling period at branch p of the TMUX, we have, $\frac{T_0 - T_1}{R_0 - R_1} = \dots = T_y$; where T_y = sampling period of $y(n)$.

In the Synthesis FB, the TMUX generates the required frequency bands through up sampling followed by R_p a parallel structure of low pass filter $G_p(z)$. The output of low

pass filter multiply with the different phase shifter to place in appropriate position in frequency spectrum. Finally, all output of multiplier is summed to form $y(n)$ for transmission [8].

$$y(n) = \sum_{p=0}^{P-1} R_p G_p(z) e^{j\omega_p n} \quad (2)$$

In the Analysis FB, to recover original signal, the received signal $\hat{y}(n)$ is first frequency shifted such that the desired signal can be processed in the baseband. Then, baseband signal pass through a low pass filter $\hat{G}_p(z)$. Output of low pass filter is down sampling by R_p is used to obtain the desired original signal which is same as input signal [8].

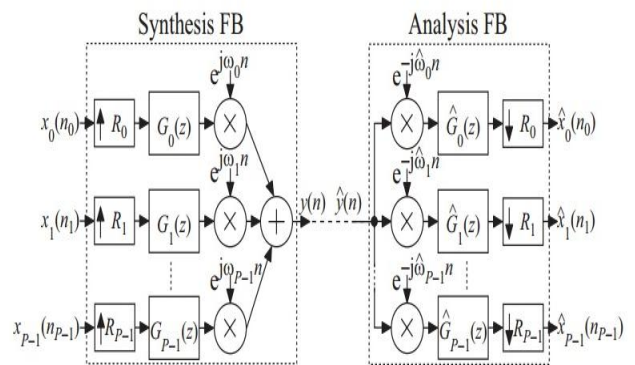


Figure 2 Multi-mode TMUX composed of Frequency shifters and SRC

$$\hat{y}(n) = \sum_{p=0}^{P-1} e^{-j\hat{\omega}_p n} \hat{G}_p(z) R_p \quad (3)$$

$G_p(z)$ is used for Sampling Rate Conversion by R_p , its polyphase representation can be written as,

$$G_p(z) = \sum_{m=0}^{R_p-1} z^{-m} G_{p,m}(z^{R_p}) \quad (4)$$

Where $G_{p,m}(z)$ = poly phase components of $G_p(z)$.

$$G_{p,m}(z) = \sum_{k=0}^L C_k(z) u_{p,m}^k \quad (5)$$

By choosing the values of $u_{p,m}$ as in (5), they possess antisymmetric according to $u_{p,m} = -u_{p,R_p-m}$.

Where,

$$u_{p,m} = -\frac{m}{R_p} + \frac{1}{2} \quad (6)$$

$G_p(z)$ is general interpolation and $\hat{G}_p(z)$ is general decimation filter of order N , it should approximate zero in the stop band and $z^{-N/2}$ in the pass band. $G_{p,m}(z)$ Should have approximate an all-pass transfer function with a fractional delay of $(\frac{N}{2} - m) / R_p$ when each term $z^{-m} G_{p,m}(z^{R_p})$ should have a delay of $z^{-N/2}$ in the pass band.

III. DCT BASED TMUX

DCT based multimode TMUX, which consists of the quadrature mirror filter (QMF) banks [4], whose magnitude response is the mirror image of that another filter which involves the splitting of an input signal into sub band and

finally reconstructs the original signal. There are two approaches for QMF design are M-channel QMF and pseudo QMF. Owing to their attractive features like free from aliasing, magnitude, amplitude distortion & phase distortion, pseudo QMF banks are of particular interest [1].

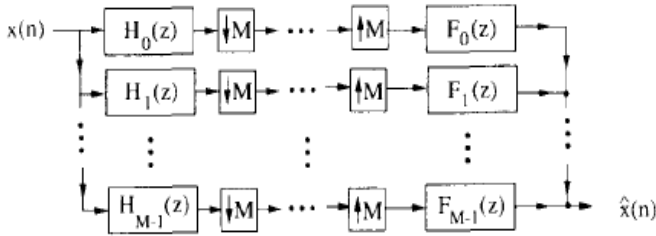


Figure 3 M-channel QMF

Fig 3 shows that , This filter bank(FB) consists of an analysis part called as analysis FB, containing filters with the transfer functions $H_k(z)$ and a synthesis part called as synthesis FB, containing filters with transfer functions $F_k(z)$; k denotes $0,1,2,\dots,M-1$. At analysis part first the incoming signal $X(z)$ is split into M frequency bands, that spitted signal passed through parallel structure of FIR filter and each sub band signal is maximally decimated by a factor of M . The M decimated signals are then processed in the synthesis bank by interpolating each signal, each interpolated signal pass through again parallel structure of FIR filter to filtering each signal and then adding the M filtered signals final output archived. $\hat{X}(n)$ is a delayed version of $X(n)$ i.e., $\hat{X}(n) = c X(n - n_0)$, $c \neq 0$, is called the perfect reconstruction property.

QMF banks, the analysis filters $H_k(z)$ and synthesis filters $F_k(z)$ are obtained by the cosine modulation of a linear phase, low-pass prototype filter,
 $H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$ (7)

As shown below [1]:

$$h_k(n) = 2h(n) \cos\left((2k + 1) \frac{\pi}{2M} \left(n - \frac{N-1}{2}\right) + \theta_k\right) \quad (8)$$

$$f_k(n) = 2h(n) \cos\left((2k + 1) \frac{\pi}{2M} \left(n - \frac{N-1}{2}\right) - \theta_k\right) \quad (9)$$

Where $0 \leq n \leq N-1$

- $h_k(n)$ = impulse response of $H_k(z)$
- $f_k(n)$ = impulse response of $F_k(z)$
- N = filter length = $2 * M * m$
- M = no. of channels
- m = arbitrary constant
- $\theta_k = (-1)^k \frac{\pi}{4}$

Poly phase implementation of modulated filter bank denotes matrix c_1 for analysis side and $(c_2)^T$ for synthesis side [3].

Where $0 \leq k \leq M-1, 0 \leq l \leq 2M-1$

$$c_1 = c_{k,q} = 2 \cos\left((2k + 1) \frac{\pi}{2M} \left(1 - \frac{N-1}{2}\right) + \theta_k\right) \quad (10)$$

$$(c_2)^T = 2 \cos\left((2k + 1) \frac{\pi}{2M} \left(2M - 1 - 1 - \frac{N-1}{2}\right) - \theta_k\right) \quad (11)$$

Then using the periodicity of the cosine modulation, we get

the relation,

$$c_{k,(l+2pM)} = (-1)^p c_{k,l} \quad (12)$$

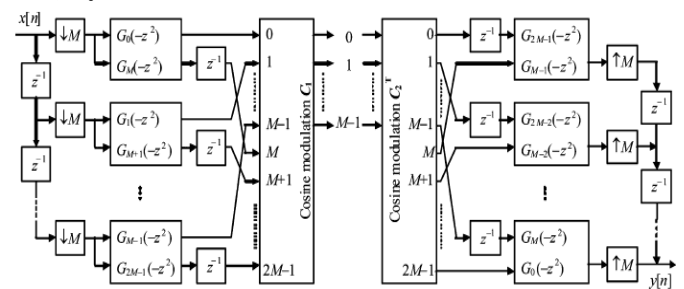


Figure 4 poly phase structure of CMFB

Obtain a poly phase structure for the analysis filter bank, first we express the prototype $H(z)$ as [1]

$$H(z) = \sum_{q=0}^{2M-1} \sum_{p=0}^{m-1} h(q + 2pM) z^{-(q+2pM)} = \sum_{q=0}^{2m-1} z^{-q} G_q(z^2) \quad (13)$$

Where $G_q(z) =$ type 1 polyphase component of $H(z)$.

Equation (7) the analysis filters can express as,

$$H_k(z) = \sum_{n=0}^{N-1} h_k(n) z^{-n} = \sum_{n=0}^{2Mm-1} h(n) c_{k,n} z^{-n} = \sum_{q=0}^{2M-1} \sum_{p=0}^{m-1} h(q + 2pM) c_{k,(q+2pM)} z^{-(q+2pM)}$$

From Equation (12)

$$H_k(z) = \sum_{q=0}^{2M-1} z^{-q} c_{k,q} \sum_{p=0}^{m-1} (-1)^p h(q + 2pM) z^{-2pM} = \sum_{q=0}^{2M-1} z^{-q} c_{k,q} G_q(-z^2) \quad (14)$$

The analysis filter bank can be expressed in matrix form as,

$$H(z) = \begin{bmatrix} H_0(z) \\ H_1(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = c_1 \begin{bmatrix} G_0(-z^2) \\ z^{-1} G_1(-z^2) \\ \vdots \\ z^{-(2M-1)} G_{2M-1}(-z^2) \end{bmatrix}$$

Where c_1 is a $M \times 2M$ cosine modulation matrix, $0 \leq k \leq M-1, 0 \leq l \leq 2M-1$.

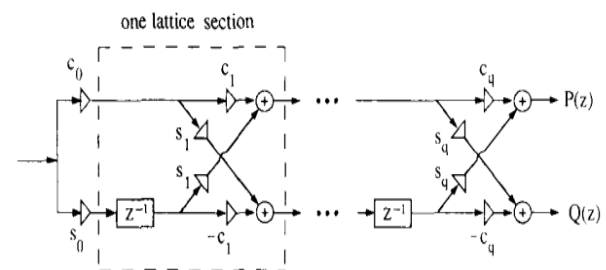


Figure 5 The q-channel lossless lattice with four-multiplier lattice sections

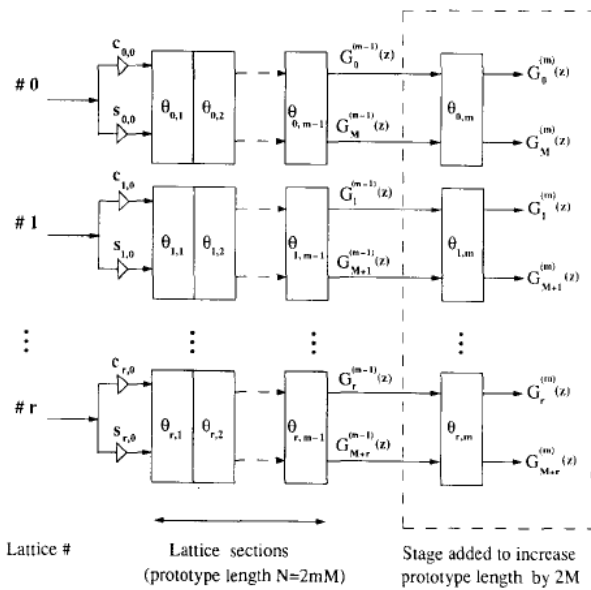


Figure 6 block diagram of the lattices used in the design of a M-channel prototype $H(z)$. The total number of lattices = $r + 1 = \lfloor \frac{M}{2} \rfloor$

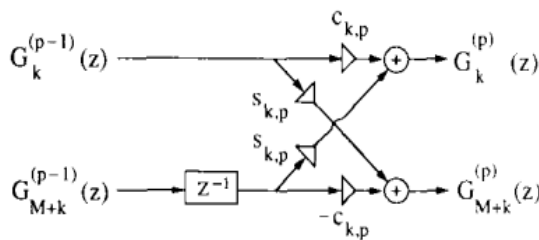


Figure 8 The two-channel lossless lattice with four-multiplier lattice sections

$$(c_k = \cos \Theta_k \text{ and } s_k = \sin \Theta_k)$$

Fig 8 shows typical four multiplier implementation of the p^{th} lattice section of k^{th} the lattice [2]. Where $p \geq 1, 0 \leq k \leq \frac{M}{2} - 1$

Simple initialization scheme for the all the parameters of the $\lfloor \frac{M}{2} \rfloor$ two channel lattices.

$$\theta_{k,p} = \begin{cases} \frac{\pi}{4}, & p = 0 \\ \frac{\pi}{2}, & 1 \leq p \leq (m - 1) \end{cases}$$

If $m > 3$ then add $(m - m_1)$ additional sections to each of the $\lfloor \frac{M}{2} \rfloor$ lattices (where $m_1 = 2$ or 3) [1].

IV. COMPARISON

All four approaches use as a TMUX in on board GEO satellite communication.

As a basic element FARROW structure based TMUX used sampling rate conversion (SRC) and DCT based TMUX used lattice structure.

Output of filter bank is given below.

Farrow based TMUX:

$$y(z) = \sum_{p=0}^{P-1} R_p G_p(z) e^{j\omega_p n}$$

DCT based TMUX:

$$Y(z) = \sum_{q=0}^{2M-1} z^{-q} c_{k,q} G_q(-z^2)$$

Bandwidth utilised in Farrow structure based TMUX is $\frac{\pi}{M}$

and DCT based TMUX utilized bandwidth is $\frac{\pi}{2M}$.

Propagation delay of Farrow structure based TMUX is M-channel and DCT based TMUX is 2M-channel.

Amplitude distortion of Farrow structure based TMUX is low. Compare to Farrow structure DCT approach based TMUX is free from amplitude distortion.

All two approaches is free from phase distortion.

Aliasing effect is present in Farrow. In DCT approach are free from aliasing.

Then we conclude that FARROW based TMUX is particularly suitable for complex signal processing and DCT is particularly used for real signal processing. Proper perfect reconstruction can only be implementing with DCT approach. Being the complex structure of Farrow based TMUX requires twice number of interconnection and routing compare to DCT approach. The design complexity is highest in FARROW compare to this lowest in DCT approach.

Free from aliasing, Free from amplitude distortion, Free from phase distortion, least complexity, least connection and for real processing is used particularly DCT approach more suitable than other approaches.

V. CONCLUSION

In this paper an implementation paper of two approaches that reduced the number of required multiplication when implementing a linear phase prototype filter of an arbitrary order used for building a perfect reconstruction filter bank has been proposed. The main purpose of this paper has been to provide an improved approach for designing M-band perfect reconstruction FIR filter banks based on lossless poly phase matrices. In this paper, we have presented derivation of necessary and sufficient condition on the poly phase components of a linear phase prototype (length $N=2Mm$, where m greater or equal to 1) such that poly phase component of the all approaches filter bank, is lossless. The detailed design procedure and complexity of implementation, FARROW structure and DCT based TMUX in this paper. This paper work for arbitrary number of channel. This paper compares FARROW and DCT approaches. This paper concludes that DCT is the best approach among the FARROW approach.

REFERENCES

- [1] R. David Koilpillai, Member, IEEE and P.P.Vidyanathan, Fellow, IEEE "Cosine-Modulated FIR Filter Banks Satisfying Perfect Reconstruction"IEEE Trans. On signal processing VOL.40, NO.4, APRIL 1992.
- [2] Oscar G. Ibaraa-Manzano and Gordana Jovanovic Dolecek, "Cosine-Modulated FIR Filter Banks Satisfying Perfect Reconstruction: An Iterative Algorithm"IEEE trans. 1999.
- [3] Robert Bregovic, Ya Jan Yu, Ari Viholainen and Yong Ching Lim "Implementation of Linear-Phase FIR Nearly Perfect Reconstruction Cosine Modulated Filter banks Utilizing Coefficient Symmetry"IEEE Trans. On circuits and systems, regular papers, VOL.-57, NO.-1, JANUARY-2010.

- [4] P.P. Vidyanathan "Quadrature Mirror Filter Bank, M-Band Extensions and Perfect-reconstruction Techniques "IEEE ASSP MAGAZINE JULY-1987.
- [5] Hakan Johanson, member IEEE and Oscar Gustafson, member, IEEE "Linear Phase FIR Interpolation, Decimation and M-band Filters Utilizing the Farrow Structure"IEEE Trans. On circuits and systems-I, regular papers, VOL.-52, NO.-10, OCTOBER-2005.
- [6] Hakan Johanson, senior member IEEE "Farrow-Structure-Based Reconfigurable Band pass Linear Phase FIR Filters for Integer Sampling Rate Convergence"IEEE Trans. On circuits and systems-II, regular papers, VOL.-58, NO.-1, JAN-2011.
- [7] S. Rajkumar, Prof. Dr. A. Sivabalan and Prof. Dr. Chandan Madzundar "FPGA Based Design of MultiMode Transmultiplexer Structure for Communication System"IOSI journal for VLSI and signal processing (IOSR-JVSP), VOL.2, Issue-5, PP 17-22, MAY-JUNE 2013.
- [8] Amir Eghbali, Hakan Johanson, and Per Lowenborg, "A Farrow-Structure-Based Multimode Transmultiplexer" IEEE Trans.- 2008.