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AN ALTERNATE METHOD TO DETERMINE THE ESSENTIAL PRIME IMPLICANTS OF A BOOLEAN FUNCTION

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Abstract: The present work proposes an alternate method to determine the essential prime implicants from a prime implicant chart derived using the tabulation method. The procedure is accomplished by assigning weights to the each of the implicants of the function. The prime implicants is visualized to group themselves in varying degrees of strengths.
Index Terms: Prime implicants, Essential Prime implicants, Quine-McCluskey method, Karnaugh Map.

I. INTRODUCTION

The classical approaches like Quine-McCluskey (QMC) method in two level logic minimization suffer from increased memory size and computation time. There are attempts to overcome the same by reducing the Prime Implicants (PI) table had been carried out by many smart algorithms. Rudell and Sangiovanni [2] proposed an alternate algorithm to QMC method. The reports suggest that 114 out of 134 benchmark functions of standard set had been solved. In the attempts to reduce the PI table, a covering algorithm Branch and Bound algorithm had been proposed. Dagenais proposed an alternate attempt referred as McBOOLE [3]. In McBOOLE, there are two tables – undecided and retained. McBOOLE is reported to be effective in 86 out of 134 benchmark functions. McBOOLE does this with the help of graphs to store the relations among the primes. Binary decision diagrams have been proposed by Coudert and Madre [4].

II. DIFFERENTIAL STRENGTH OF ADHESION

The minimization of logic function is visualized as a problem of differential strength of adhesion among the prime implicants. The prime implicants with strength more than the threshold is deemed as essential prime implicant.

Associated Weights: The proposed method of determining the minimum sum-of-product expression is accomplished by assigning 'weights' to all the prime implicants obtained after the first step of Quine-McCluskey method. The number of literals which make up the prime implicant determine the 'weights' associated with each prime implicant. The number of literals depends on the number of 1s being grouped which varies as the power of 2. A single 1 or a group of two adjacent 1s or four adjacent 1s or a maximum of eight adjacent 1s form a prime implicant in a four variable problem. Each of the prime implicants obtained at the end of the first step of Quine-McCluskey method clearly shows the number of 1s forming the particular prime implicant. The proposed method has an associated 'weight' with each of the prime implicant. It is assumed that all the prime implicant will have a maximum weight of '1' (not a logical one). The maximum weight of '1' is distributed equally to all the constituents of the prime implicant when considered alone. If two adjacent 1s are grouped together to form a prime implicant, each of the two 1s is assigned an equal weight of 0.5 each. If the constituents 1s of the group are four in number, then each 1s in the group is assigned 0.25 each. Karnaugh map is shown in Figure 1 and Table 1 shows the associated weights of the prime implicants.

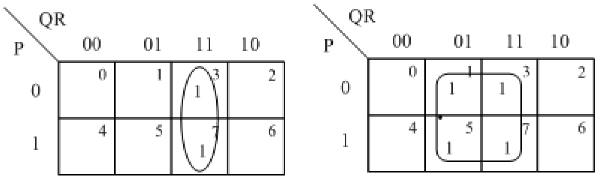

Figure 1: Karnaugh maps of groupings of 1s

Table 1(a): Associated weights of Implicants

Strengths	f	3	7
Initial		1	1
(3,7)	2	0.5	0.5
Final		0.5	0.5

Table 1(b): Associated weights of Implicants

Strengths	f	1	3	5	7
Initial		1	1	1	1
(1, 3, 5, 7)	4	0.25	0.25	0.25	0.25
Final		0.25	0.25	0.25	0.25

If more than one prime implicants overlap, then the associated weight of that particular 1 (implicant) will share or lose half of its weight to the adjacent group. Figure. 2 shows the Karnaugh map. The associated weights are shown in Table 2.

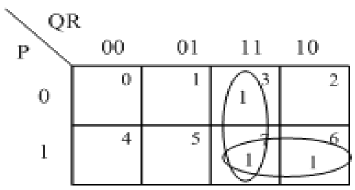

Figure 2: Two loops sharing the same 1

Table 2:Associated weights of Implicants

Strengths	F	3	6	7
Initial		1	1	1
(3,7)	2	0.5	-	0.5
(6,7)	2	-	0.5	0.25
Final		0.5	0.5	0.25

B. Removal of groupings with lower strengths: The assignment of weights is continued for all the prime implicants in the prime implicants chart. The prime implicants with more number of 1s is given priority ahead of the prime implicants made up of lesser number of 1s. The final combined weights of each of the prime implicant is assessed based on the threshold. The prime implicants lesser than or equal to the threshold are discarded and the ones which have higher values are the essential prime implicants. The threshold value varies according to the problem in hand. The maximum value assigned to an implicant in the problem is taken as the threshold i.e., if the problem in hand has prime implicants with two 1s, then the threshold is 0.5. If the problem has only prime implicants with four 1s, then the threshold is 0.25 and so on.

Illustrative Examples: The list of prime implicants at the end of the first step of Quine-McCluskey method for the problem f = Em (0, 1, 2, 5, 6, 7, 8, 9, 10, 14) are (0, 1, 8, 9), (0, 2, 8, 10), (2, 6, 10, 14), (1, 5), (5, 7), (6, 7). The threshold is 0.5 for the problem in hand as the maximum 'weight' taken by the implicant is 0.5 as shown in Table 3.

Table 3: Threshold for the Prime Implicants of Em = (0,1,2,5,6,7,8,9,10,14)

Group	f = No. of 1s	1/f
(0, 1, 8, 9)	4	0.25
(0, 2, 8, 10)	4	0.25
(2, 6,10, 14)	4	0.25
(1, 5)	2	0.50
(5, 7)	2	0.50
(6, 7)	2	0.50

Table 4: Changing weights of each of the implicants of Em = (0,1,2,5,6,7,8,9,10,14)

Strengths	0	1	2	5	6	7	8	9	10	14
Initial	1	1	1	1	1	1	1	1	1	1
(0, 1, 8, 9)	.25	.25	-	-	-	-	.25	.25	-	-
(0, 2, 8, 10)	.1250	-	.25	-	-	-	.1250	-	.25	-
(2, 6, 10, 14)	-	-	.1250	-	.25	-	-	.25	.1250	.25
(1, 5)	-	.1250	-	.50	-	-	-	-	-	-
(5, 7)	-	-	-	.25	-	.50	-	-	-	-
(6, 7)	-	-	-	-	.1250	.25	-	-	-	-
Final	.1250	.1250	.1250	.25	.1250	.25	.1250	.25	.1250	.25

Table. 4 shows the weights associated with each implicants in the PIs taken and the final values taken by them after groupings. The final values of the individual groupings are added to arrive at the combined strength of the groupings. The order in which the groupings are taken is random.

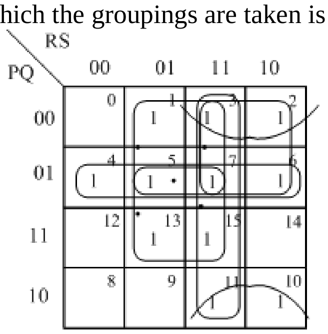

Figure 4: PIs and EPIs of Em = (1,2,3,5,6,7,10,11,13,15)

Table. 5 shows that four of the prime implicants (0, 2, 8, 10), (1, 5), (5, 7), (6, 7) are very weak. This is because the combined strengths of the four prime implicants are less than the threshold. Of the four PIs, two PIs with four 1s and of the 3 PIs with two 1s, the weakest among them is eliminated. The PIs (1, 5) and (6, 7) are the weakest as shown in the Table. 5.

Elimination of (1, 5) makes the PI (0, 1, 8, 9) to increase from its strength from .625 to .75 as implicant 1 in (1,5) gives its share of 0.125 to implicant 1 in (0, 1, 8, 9). The same is true with (5, 7) whose strength increases to 0.5 to .75. Elimination of (6, 7) makes the PI (2, 6, 10, 14) to have an increased strength of 0.75 and PI (5, 7) to become 1.0

Now the weakest PI with four 1s is chosen for elimination. The process gives PI (0, 1, 8, 9) a strength of 1.0 and PI (2, 6, 10, 14) a strength of 1.0.

Table 5: Combined weights of the Prime Implicants of Em = (0,1,2,5,6,7,8,9,10,14)

Prime Implicants	Combined Strength	Total	Status
(0, 1, 8, 9)	.1250 + .1250 + .1250 + .25	.625	Strong
(0, 2, 8, 10)	.1250 + .1250 + .1250 + .1250	.5	Weak
(2, 6,10, 14)	.1250 + .1250 + .1250 + .25	.625	Strong
(1, 5)	.1250 + .25	.375	Weak
(5, 7)	.25 + .25	.5	Weak
(6, 7)	.1250 + .25	.375	Weak

Table 6: Restored weights of the Prime Implicants of Em = (0,1,2,5,6,7,8,9,10,14)

Prime Implicants	Combined Strength	Total	Status
(0, 1, 8, 9)	.1250 + .25 + .1250 + .25	.75	Strong
(0, 2, 8, 10)	.1250 + .1250 + .1250 + .1250	.5	Weak
(2, 6,10, 14)	.1250 + .25 + .1250 + .25	.75	Strong
(5, 7)	.50 + .50	1.0	Strong

Figure. 3 and Table. 7 show the PIs and EPIs for the problem f = Em (0, 1, 2, 5, 6, 7, 8, 9, 10, 14).

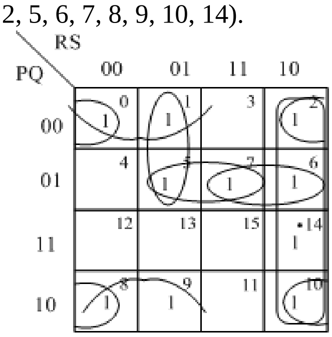

Figure 3(a): Prime Implicants of Em = (0,1,2,5,6,7,8,9,10,14)

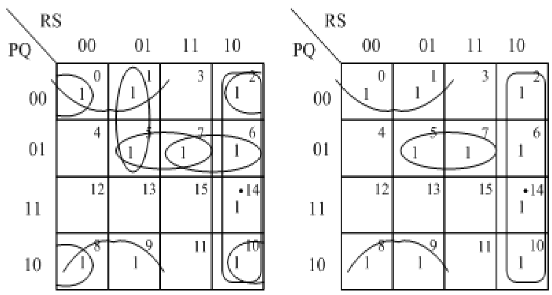

Figure 3(b): PIs and EPIs of Em = (0,1,2,5,6,7,8,9,10,14)

Table 7: EPIs of Em = (0,1,2,5,6,7,8,9,10,14).

Prime Implicants	Combined Strength	Total	Status
(0, 1, 8, 9)	.25 + .25 + .25 + .25	1.0	Essential
(2, 6,10, 14)	.25 + .25 + .25 + .25	1.0	Essential
(5, 7)	.50 + .50	1.0	Essential

The problem defined by the sum of minterms given as f = Em (1, 2, 3, 4, 5, 6, 7, 10, 11, 13, 15) has six prime implicants from the first step of Quine-McCluskey method (Figure. 4). All the six PIs are essential in this problem which has a threshold of 0.25.

III. CONCLUSION

Thus an alternative method to determine the Essential Prime Implicants from a Prime Implicant chart is proposed with this paper. However the Prime Implicant chart is taken from the conventional techniques. The method to extend the determination of Prime Implicant also based on the differential strength of adhesion is foreseen as a future development.

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