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VIBRATION ANALYSIS OF COMPOSITE BEAM

Sajid Ahmad Shah[1], Er. Manni Sharma[2], Dr, Pooja Sharma[3]
[1]M. Tech Scholar, [2,3]Assistant Professor & HOD (Desh Bhagat University),

Abstract: Composite beams and beam like elements are principal constituents of many structures and used widely in high speed machinery, aircraft and light weight structures. Crack is a damage that often occurs on members of structures and may cause serious failure of the structures. The influence of cracks on dynamic characteristics like natural frequencies, modes of vibration of structures has been the subject of many investigations. However, the parametric studies like effect of geometry, crack location and support conditions on natural frequencies of composite beam are scarce in literature. In the present work, a numerical study using finite element is performed to investigate the free vibration response of composite beams. The finite element software ANSYS is used to simulate the free vibrations. A variety of parametric studies are carried out to see the effects of various changes in the laminate parameters on the natural frequencies. The parameters investigated include the effects of fiber orientation, the location of cracks relative to the restricted end, depth of cracks, volume fraction of fibers, length of beam and support conditions. The study shows that the highest difference in frequencies occur when the value of the fiber orientation equal to zero degree. The increase of the beam length results in a decrease in the natural frequencies of the composite beam and also shows that an increase of the depth of the cracks leads to a decrease in the values of natural frequencies.

I. INTRODUCTION

In the recent decades, fiber reinforced composite materials are being used more frequently in many different engineering fields. The automobile, aerospace, naval, and civil industries all use composite materials in some way. Composite materials are gaining popularity because of high strength, low weight, resistance to corrosion, impact resistance, and high fatigue strength. Other advantages include ease of fabrication, flexibility in design, and variable material properties to meet almost any application. Beams and beam like elements are principal constituents of many mechanical structures and used widely in high speed machinery, aircraft and light weight structures. Fiber-reinforced laminated beams constitute the major category of structural members, which are widely used as movable to elements, such as robot arms, rotating machine parts, and helicopter and turbine blades. Similar to other structural components, beams are subjected to dynamic excitations. Reducing the vibration of such structures is a basic requirement of engineers. One method to reduce the vibration of a structure is to move its natural frequencies away from frequency of excitation force. There are different methods to modify the natural frequencies of beam structures. In general, any continuous structure has infinite degrees of freedom and, consequently, an infinite number of natural frequencies and the corresponding modal shapes. If a structure vibrates with a frequency equal to a natural one, the vibration amplitude grows rapidly with time, requiring a very low input energy. As a result, the structure either fails by overstressing, or the nonlinear effects limit the amplitude to a large value, leading to high-cycle fatigue damage. Thus, for any structure, its natural frequencies must be determined in order to ensure that the loading frequencies imposed and the natural frequencies differ considerably; in other words, to avoid resonances.

To avoid structural damages caused by undesirable vibrations, it is important to determine:

1 - Natural frequencies of the structure to avoid resonance;

2 - Mode shapes to reinforce the most flexible points or to determine the right positions to reduce weight or to increase damping;

3 - Damping factors.

With respect to these dynamic aspects, the composite materials represent an excellent possibility to design components with requirements of dynamic behavior (Tita, 2003).

During operation, all structures are subjected to degenerative effects that may cause initiation of structural defects such as cracks which, as time progresses, lead to the catastrophic failure or breakdown of the structure. Thus, the importance of inspection in the quality assurance of manufactured products is well understood. 3

Cracks or other defects in a structural element influence its dynamical behavior and change its stiffness and damping properties. Consequently, the natural frequencies of the structure contain information about the location and dimensions of the damage (Krawczuk, 1995). Structural damage detection has gained increasing attention from the scientific community since unpredicted major hazards, most with human losses, have been reported. Aircraft crashes and the catastrophic bridge failures are some examples. Development of an early damage detection method for structural failure is one of the most important keys in maintaining the integrity and safety of structures. The cracks can be present in structures due to their limited fatigue strengths or due to the manufacturing processes. These cracks open for a part of the cycle and close when the vibration reverses its direction. These cracks will grow over time, as the load reversals continue, and may reach a point where they pose a threat to the integrity of the structure. As a result, all such structures must be carefully maintained and more generally, SHM denotes a reliable system with the ability to detect and interpret adverse “change” in a structure due to damage or normal operation (Ramanamurty 2008).

II. LITERATURE REVIEW

Nikpur and Dimarogonas (1988) presented the local compliance matrix for unidirectional composite materials. They have shown that the interlocking deflection modes are enhanced as a function of the degree of anisotropy in composites. 7

QIAN and Gu (1990) derived an element stiffness matrix of a beam with a crack from an integration of stress intensity factors, and then a finite element model of a cracked beam is established. This model is applied to a cantilever beam with an edge-crack, and the eigen frequencies are determined for different crack lengths and locations. Finally, a simple and direct method for determining the crack position, based on the relationship between the crack and the eigen couple (eigenvalue and eigenvector) of the beam, is proposed and this method can be suggested to complex structures with various cracks, if their stress intensity factors are known.

Ostachowicz and Krawczuk (1991) presented a method of analysis of the effect of two open cracks upon the frequencies of the natural flexural vibrations in a cantilever beam. Two types of cracks were considered: double-sided, occurring in the case of cyclic loadings, and single-sided, which in principle occur as a result of fluctuating loadings. It was also assumed that the cracks occur in the first mode of fracture: i.e., the opening mode. An algorithm and a numerical example were included.

Manivasagam and Chandrasekaran (1992) presented results of experimental investigations on the reduction of the fundamental frequency of layered composite materials with damage in the form of cracks.

Krawczuk (1994) formulated a new beam finite element with a single non-propagating one-edge open crack located in its mid-length for the static and dynamic analysis of cracked composite beam-like structures. The element includes two degrees of freedom at each of the three nodes: a transverse deflection and an independent rotation respectively. He presented the exemplary numerical calculations illustrating variations in the static deformations and a fundamental bending natural frequency of a composite cantilever beam caused by a single crack.

Krawczuk and Ostachowicz (1995) investigated Eigen frequencies of a cantilever beam made from graphite-fiber reinforced polyimide, with a transverse on-edge non-propagating open crack. Two models of the beam were presented. In the first model the crack was modeled by a massless substitute spring Castigliano"s theorem. The second model was based on the finite element method. The undamaged parts of the beam were modeled by beam finite elements with three nodes and three degrees of 8 freedom at the node. The damaged part of the beam was replaced by the cracked beam finite element with degrees of freedom identical to those of the non-cracked done. The effects of various parameters the crack location, the crack depth, the volume fraction of fibers and the fibers orientation upon the changes of the natural frequencies of the beam were studied.

Ghoneam (1995) presented the dynamic characteristics laminated composite beams (LCB) with various fiber orientations and different boundary fixations and discussed in the absence and presence of cracks. A mathematical model was developed, and experimental analysis was utilized to study the effects of different crack depths and locations, boundary conditions, and various code numbers of laminates on the dynamic characteristics of CLCB. The analysis showed good agreement between experimental and theoretical results.

Dimarogonas (1996) reported a comprehensive review of the vibration of cracked structures. This author covered a wide variety of areas that included cracked beams, coupled systems, flexible rotors, shafts, turbine rotors and blades, pipes and shells, empirical diagnoses of machinery cracks, and bars and plates with a significant collection of references.

Krawczuk, Ostachowicz and Zak (1997) presented a model and an algorithm for creation of the characteristic matrices of a composite beam with a single transverse fatigue crack. The element developed had been applied in analyzing the influence of the crack parameters (position and relative depth) and the material parameters (relative volume and fiber angle) on changes in the first four transverse natural frequencies of the composite beam made from unidirectional composite material.

Chondros (1998) developed a continuous cracked beam vibration theory for the lateral vibration of cracked Euler Bernoulli beams with single edge or double edge open cracks. The HuWashizuBarr variational formulation was used to develop the differential equation and the boundary conditions of the cracked beam as a one dimensional continuum. The displacement field about the crack was used to modify the stress and displacement field throughout the bar. The crack was modeled as a 9 continuous flexibility using the displacement field in the vicinity of the crack found with fracture mechanics methods. Hamada (1998) studied the variations in the Eigen-nature of cracked composite beams due to different crack depths and locations. A numerical and experimental investigation has been made. The numerical finite element technique was utilized to compute the Eigen pairs of laminated composite beams through several states of cracks. The model was based on elastic-plastic fracture mechanics techniques in order to consider the crack tip plasticity in the analysis. The model has been applied to investigate the effects of state of crack, lamina code number, boundary condition on the dynamic behavior of composite beams. Zak, Krawczuk and Ostachowicz (2000) developed the work models of a finite delaminated beam element and delaminated plate element. They carried out an extensive experimental investigation to establish changes in the first three bending natural frequencies due to delamination. The subsequent results of the numerical calculations were consistent the results of the experimental investigations.

Banerjee (2001) derived exact expressions for the frequency equation and mode shapes of composite Timoshenko beams with cantilever end conditions in explicit analytical form by using symbolic computation. The effect of material coupling between the bending and torsional modes of deformation together with the effects of shear deformation and rotatory inertia is taken into account when formulating the theory. The expressions for the mode shapes were also derived in explicit form using symbolic computation.

III. THE METHODOLOGY

- Modal analysis of ANSYS is used to determine the natural frequencies and mode shapes, which are important parameters in the design of a structure for dynamic loading conditions. They also required for spectrum analysis or for a mode superposition harmonic transient analysis.
- Modal analysis in ANSYS program is linear analysis.
- The damped and QR damped methods allow to include damping in the structure.

GOVERNING EQUATION

The differential equation of the bending of a beam with a mid-plane symmetry (B*ij* = 0) so that there is no bending-stretching coupling and no transverse shear deformation (εxz=0) is given by;

(1)

It can easily be shown that under these conditions if the beam involves only a one layer, isotropic material, then I*S11 = EI = Ebh3/12* and for a beam of rectangular cross-section Poisson‟s ratio effects are ignored in beam theory, which is in the line with Vinson & Sierakowski (1991).

In Equation 1, it is seen that the imposed static load is written as a force per unit length. For dynamic loading, if Alembert‟s Principle are used then one can add a term to Equation.1 equal to the product mass and acceleration per unit length.

BEAM MODEL

The model chosen is a cantilever composite beam of uniform cross-section A, having an open transverse crack of depth „a‟ at position L1. The width, length and height of the beam are B, L and H, respectively in Fig. 3.1. The angle between the fibers and the axis of the beam is α.

Fig. 3.1 Schematic diagram cantilever composite beam with a crack

MODELLING PROCEDURE IN ANSYS 13

Regardless of the type of problem involved, an ANSYS analysis consists of the same steps as follows: 16

1. Preprocessing
2. Solution stage
3. Post processing.

After selecting the type of analysis in the preferences, the next step in the preprocessing is to choose an element type. The element type includes a list of general categories such as Structural Mass, Structural Link, Structural Solid, Beam, Solid Sell etc. A number of different specific elements will appear for each general category. Each element has its own set of DOFs, which are the degrees of freedom for which ANSYS will find a solution. Next material properties, real constants, section etc. need to input.

The modeling phase entails geometry definition. This is where you draw a 2D or 3D representation of the problem. ANSYS has a very powerful modeler built into the preprocessor.

The modeler allows the user to construct surfaces and solids to model a variety of geometries. For any given geometry, there are often several different ways to create the model. Before the meshing phase you will define material properties and choose a finite element suitable for the problem. In the meshing phase the model discretized i.e. creating the mesh.

In the solution phase, boundary conditions and loads need to be defined. The types of loads and boundary conditions you select depend on the simplifications being made. ANSYS will then attempt to solve the system of equations defined by the mesh and boundary conditions.

Finally, when the solution is complete, you will need to review the results using the post processor. The ANSYS post processor provides a powerful tool for viewing results .These results may be color contour plots, line plots, or simply a list of DOF results for each node. 17

MESHING:

Meshing a model can be the most difficult part of using any finite element package. While ANSYS gives the user a variety of automatic options so far as meshing is concerned, you are urged to use caution when using these tools. It is usually best to think about how you would like to mesh your model before you even go about making a model and creating areas. In general, ANSYS has two methods of meshing:

1. Free meshing

The free mesh has no recognizable pattern and no regularity in the element shapes. Free meshing is easy but for complex geometries can often lead to distorted elements that undermine accuracy. Too often users free mesh a model because it is easy without bothering to worry about the resulting mesh. Free meshing is available for 2D quadrilateral and triangular element shapes. Free meshing can only produce 3D tetrahedral elements for solid models.

2. Mapped meshing.

Mapped meshes are easier to control and are oftentimes more accurate. Mapped meshes allow the user to more carefully specify the size and shape of the mesh in local regions. Mapped meshing is available for 2D and 3D elements.

Mapped meshes are controlled by specifying element divisions on boundaries and by splitting areas and volumes in certain ways. Once you have split the areas and/or volumes in accordance with the above rules, use lsel to select the lines and lesize to set the number of element divisions along that line. The feature of mapped meshing allows the user to place smaller elements in the areas of high stress gradient (near the crack) while using larger elements where the gradient is not so steep. 19

There are restrictions to the use of mapped meshing;

For 2D element, each area must be four-sided i.e. be made up of four lines. If the area is made up of more lines, you will need to split up the area to create sub-areas with four sides or you must concatenate lines so that four lines define the area.

For 3Delement, each volume must have 6 faces (6 bounding areas). You will need to split volumes or concatenate lines and areas to create 6-faced volumes.

MODELING OF COMPOSITE BEAM USING ANSYS 13

Firstly it is required to give preference for what type analysis you want to do, here we are analyzing for beam so structural part is selected as Shown in fig.

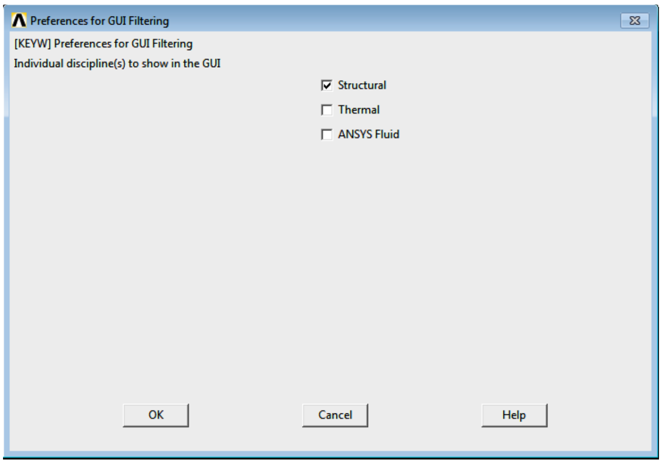
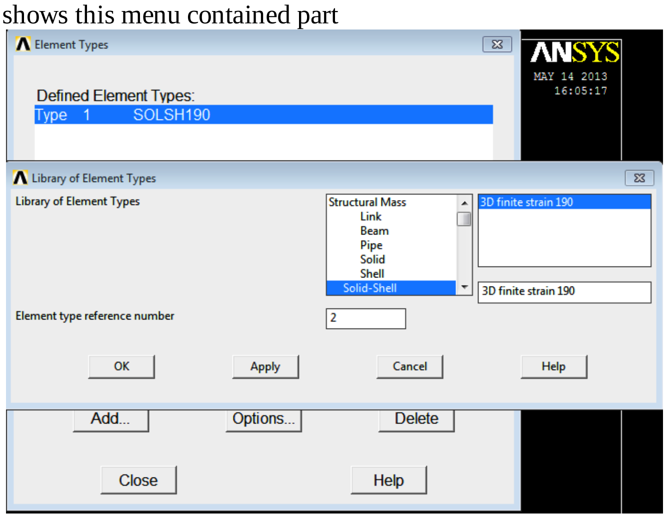


Fig Preference menus on ANSYS

Preprocessor

Now next step is preprocessor, where the preprocessor menu basically used to inputs the entire requirement thing for analysis such as- element type, real constraints, material properties, modeling, meshing, and loads. Element menu contains – defined element type and degree of freedom defined where we have to give the element structure type such as BEAM, SOLID, SHELL, and the degree of freedom , here in this analysis selected part is SOLID SHELL190 fig . shows this menu contained part



Solution Stage

The default direct frontal solver is fine for small linear problems. However, the size limitations become obvious when the user attempts to solve large 3D problems. Solving the FE problem is tantamount to solving a matrix equation with a very large matrix. Iterative methods are generally faster for bigger problems. ANSYS provides several different solver options, each of which may be more or less appropriate for a given problem.

Before going to solve the problem, we have to introduce the analysis type i.e., static analysis, harmonic analysis, or modal analysis etc. to the problems. We have selected modal analysis for our problem, because we want frequencies and mode shapes as output.

Following steps are to be followed for type of analysis:

Go to Preprocessor □Loads □ Analysis type □ New analysis.

Click Modal in the type of analysis box.

Go to Analysis options □ Mode extraction method □ Block Lanczos .

Enter the Number of Modes required and click Ok

Post Processing

The ANSYS post processor provides a powerful tool for viewing results. We can see the following results in Post processing:

1. Result summary
2. Failure criteria
3. Plot results
4. List results
5. Nodal calculation.

The procedure for vibration analysis of composite beam with crack, i.e., V-notch is same as the above described steps, but a little bit modification in modelling process. Crack is created in the composite beam using key points and lines to define areas. Volumes can be made from extruding the areas and then using Boolean operation to achieve a crack in a composite beam. For proper meshing, we will divide the beam at location of crack into two volumes using working plane.

NUMERICAL RESULTS

After obtaining the comparison with previous study with the existing literatures, the results for various parametric studies like effect of geometry, crack location and support conditions on natural frequencies of composite beam are presented. The changes of the two first natural frequencies of the beam due to the crack as functions of fiber volume fraction are analyzed. Similarly, the three first natural frequencies of the composite beam due to the crack as functions of fiber orientations (α) and fiber volume fractions are analyzed for free vibration of a composite beam with multiple cracks for different crack positions. The beam assumed to be made of unidirectional graphite fiber-reinforced polyamide. The geometrical characteristics of the graphite fiber-reinforced polyamide composite beam are chosen as the same of those used in Krawczuk & Ostachowicz (1995). The material properties of the graphite fiber-reinforced polyamide composite are taken as below:

E11 = 139.18GPa,
E22 = 8.0539GPa,
G12 = 3.0352GPa,
G23 = 2.9944GPa,
ν12 = 0.2650,
ν23 = 0.3448, 31

The geometrical characteristics, the length (L), height (H) and width (B) of the composite beam were chosen as 1.0m, 0.025 and0.050 m, respectively.

(A) Vibration Analysis of composite beam with single crack

Effect of various parameters like volume fraction of fibers, length of the beam, boundary conditions and crack locations of cracked composite beam on first, second and third non-dimensional natural frequencies studied and explained as below.

Effect of Volume Fraction of fiber on Natural frequencies
First non-dimensional natural frequencies of the cracked composite beam as a function of volume fraction of fibers V for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (angle of fibers α = 0 degree and crack location L1/L= 0.1)

Angle of fibers(degree)	Volume fraction of fiber	First Non-dimensional Frequency	
		Relative crack depth	Relative crack depth
0	0	1.7749	1.6767
	0.1	1.7437	1.5542
	0.3	1.7107	1.4945
	0.5	1.7090	1.4912
	0.7	1.7254	1.5189
	0.9	1.7653	1.5943
	1	1.8049	1.7019

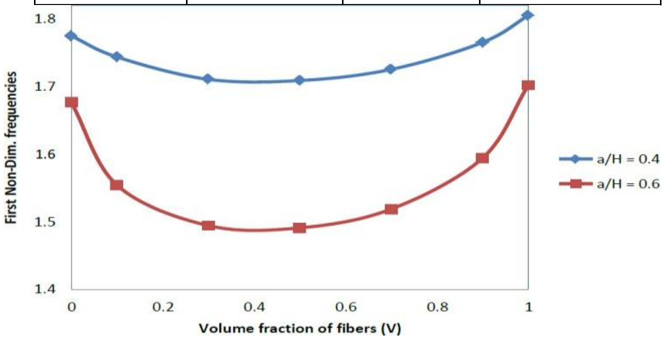


Table Second non-dimensional natural frequencies of the cracked composite beam as a function of volume fraction of fibers V for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (angle of fibers α = 0 degree and crack location L1/L= 0.1)

Angle of fibers(degree)	Volume fraction of fiber	Second Non-dimensional Frequency	
		Relative crack depth	Relative crack depth
0	0	4.5167	4.4458
	0.1	4.5356	4.4402
	0.3	4.4851	4.3930
	0.5	4.4828	4.3911
	0.7	4.5110	4.4168
	0.9	4.5690	4.4745
	1	4.5969	4.5240

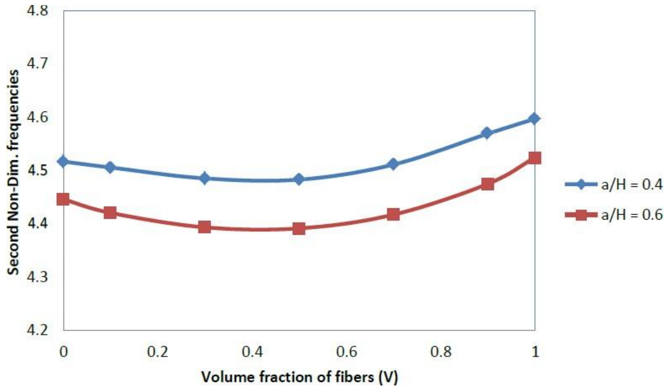


Fig. Second non-dimensional natural frequencies of the cracked composite beam as a function of volume fraction of fiber V for different values of the crack depth a/H = 0.2, 0.4 and 0.6 (angle of fibers α = 0 degree, crack location x/L = 0.1)

Fig. presents the influence of the volume fraction of fibers on the first two non- dimensional natural frequencies for different values of the crack depth ratios (a/H). Here various values of the fiber volume fraction have been considered to study its effect on first and second non dimensional frequencies. The angle of fiber is taken as zero degree and crack location is at a distance L1= 0.1L (m) from the fixed end of the beam. The flexibility due to crack is high when the volume fraction of the fiber is between 0.2 and 0.8 and maximum when the fiber fractions is nearly 0.45. This is due to the fact that the flexibility of the composite beam due to crack is a function of fiber volume fraction. Therefore, if the fiber volume fraction is between 0.2 and 0.8 and crack depth ratio is getting higher, the frequency reductions are relatively high as observed in above graphs.

Table shows Second non-dimensional natural frequencies of the composite beam with crack as a function of crack location (L1/L) for different crack depth ratios (for volume fraction of fibers V = 0.5

Angle of fibers(degree)	Crack location (L1/L)	Second Non-dimensional Frequency		
		Relative crack depth	Relative crack depth	Relative crack depth
0	0	4.5871	4.5871	4.5871
	0.1	4.5537	4.4828	4.3911
	0.3	2.2957	2.2867	2.2757
	0.5	4.0722	4.0399	3.987
	0.7	2.3407	2.3156	2.298
	0.9	4.5862	4.5839	4.5727

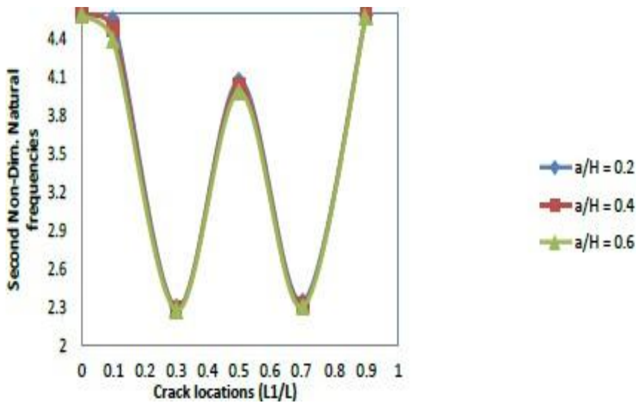


Fig Second non-dimensional natural frequencies of the cracked composite beam as a function of different crack location for different crack depth ratios. (for V = 0.5)

From Fig., when V_f = 0.5, a/H = 0.2, 0.4 and 0.6 with crack locations (L1/L) from 0 to 0.9 for cantilever composite beam, the first natural frequency is maximum at crack locations L1/L = 0.1 and L1/L = 0.9. The natural frequency decreases from crack location L1/L = 0.1 up to the minimum value at crack location L1/L = 0.5 and then increases to the maximum value at crack location L1/L = 0.9. The second natural frequency as shown in Figure 4.14 is maximum at crack locations L1/L =
0.1 and L1/L = 0.9 and minimum at crack locations L1/L = 0.3 and L1/L = 0.7.The natural frequency decreases from crack location (L1/L) of zero up to the minimum value at crack locationL1/L = 0.3 and then increases to the maximum value at crack location L1/L = 0.5. The curve is symmetric around the middle crack position (L1/L
= 0.5). As the crack depth increases, the corresponding natural frequencies decrease for each crack location. This is compatible with the increase of flexibility or decrease in the stiffness of the beam.

(B) Vibration analysis of composite beam with multiple cracks

The effects of various parameters on the vibration of composite beam with multiple cracks are presented below. The Finite element analysis is carried out for free vibration of a composite cracked beam for various crack locations and crack depth ratio a/H = 0.4 for the example problem considered by Kisa (2003). In Fig.
, the variations of the first three lowest natural frequencies of the composite beam with multiple cracks are shown as a function of fiber orientation (α) for different cracks locations. In these figures three cases, labeled as E, F and G, were considered .The cracks locations (L1/ L, L2 /L, L3/L) for the cases E, F and G, where chosen as (0.05, 0.15, 0.25), (0.45, 0.55, 0.65), (0.75, 0.85, 0.95) respectively. The non-dimensional natural frequencies are normalized according to Eq. (7).

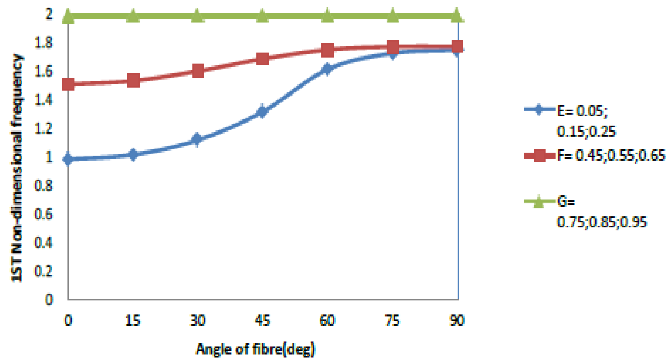


Fig The first non-dimensional natural frequencies as a function of angle of fibers for the cases of three cracks located differently, as indicated a/H=0.4 and V=0.5

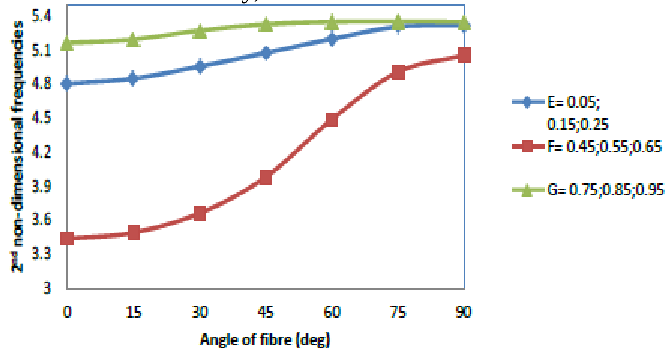


Fig The Second non-dimensional natural frequencies as a function of angle of fibers for the cases of three cracks located differently, as indicated a/H=0.4 and V=0.5

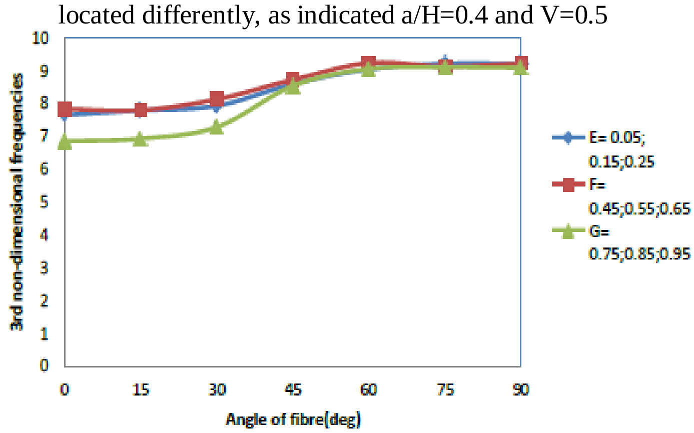


Fig The third non-dimensional natural frequencies as a function of angle of fibers for the cases of three cracks located differently, as indicated a/H=0.4 and V=0.5

It can be clearly seen from the Figs. that, when the cracks are placed near the fixed end the decrease in the first natural frequencies are highest, whereas, when the cracks are located near the free end, the first natural frequencies are almost unaffected. This observation goes to the conclusion that, the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively.

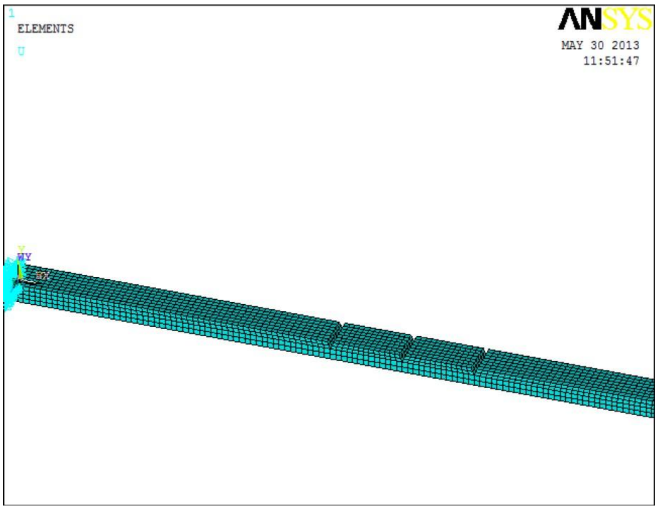


Fig. Cantilever composite beam with multiple cracks modeled in ANSYS 13

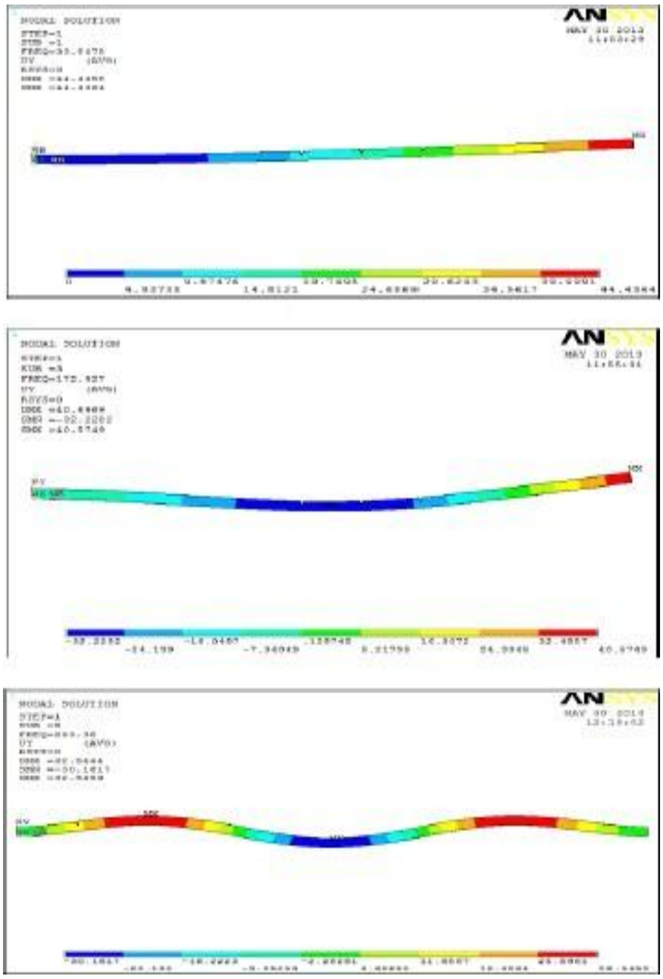


Fig. First three mode shapes of clamped-free composite beam with multiple cracks for crack locations at L1/L= 0.45, L2/L = 0.55 and L3/L = 0.65.

IV. CONCLUSION

The following conclusions can be drawn from the present investigations of the composite beam finite element having transverse open crack i.e. v-notch. This element is versatile and can be used for static and dynamic analysis of a composite beam.

- The in-plane bending frequencies decrease, in general, as the fiber angle increases; the maximum occur at α = 0°and decrease gradually with increasing the fiber angle up to a minimum value obtained for α =90°.
- In case of composite beam with crack, as the angle of fibers (α) increases the value of the natural frequencies also increases. The most difference in frequency occurs when angle of fibers is zero degree.
- The non-dimensional natural frequencies is also depends upon the volume fraction of the fibers. The flexibility due to crack is high when the volume fraction of the fiber is between 0.2 and 0.8 and maximum when the fiber fractions is nearly 0.45
- Decrease in the natural frequencies become more intensive with the growth of the depth of crack.
- The increase of the beam length results in a decrease in the natural frequencies of the composite beam
- Boundary conditions have a remarkable influence on the natural frequencies. The natural frequencies for the clamped-clamped support are higher compared to clamped- free support condition.
- The first natural frequency is maximum at crack locations L1/L = 0.1 and L1/L = 0.9 and minimum at L1/L = 0.5. While the second natural frequency is minimum at crack locations L1/L = 0.3 and L1/L = 0.7.
- The effect of cracks is more pronounced near the fixed end than at far free end. It is concluded that the first, second and third natural frequencies are most affected when the cracks located at the near of the fixed end, the middle of the beam and the free end, respectively

SCOPE FOR FUTURE WORK

1. The vibration results obtained using ANSYS 13 can be verified by conducting experiments.
2. The dynamic stability of the composite beam with cracks
3. Static and dynamic stability of reinforced concrete beam with cracks.
4. The Vibration analysis of composite beam by introducing inclined cracks in place of transverse crack.

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