ANALYSIS ON DYNAMIC BEHAVIOR OF COMPOSITE BEAMS WITH DIFFERENT CROSS SECTION

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Abstract; The increasing use of composite materials across various fields such as aerospace, automotive, civil, naval and other high performance engineering applications are due to their light weight, high specific strength and stiffness, excellent thermal characteristic, ease in fabrication and other significant attributes. The present study deals with experimental investigation on free vibration of laminated composite beam and compared with the numerical predictions using finite element method (FEM) in ANSYS environment. A program is also developed in MATLAB environment to study effects of different parameters. The scope of the present work is to investigate and understand the effect of different parameters including cross sectional shape on modal parameters like modal frequency, mode shapes. Experimental investigation is carried out by Impulsive frequency response test under fixed- free and fixed-fixed boundary conditions. Composites Beams are fabricated using woven glass fabric and epoxy by hand layup technique. Modal analysis of various cross sectional beams were reported, compared and discussed. The finite element modeling has been done by using ANSYS 14 and compared with the experimental results. Two-node, finite elements of three degrees of freedom per node and rectangular section are presented for the free vibration analysis of the laminated composite beams in this work. The effects of different parameters including ply orientation, number of layers, effect of the length of the beam and various boundary conditions of the laminated composite beams are discussed.

I. INTRODUCTION

The widespread use of composite structures in aerospace applications has stimulated many researchers to study various aspects of their structural behavior. These materials are particularly widely used in situations where a large strength-to-weight ratio is required. Similarly to isotropic materials, composite materials are subjected to various types of damage, mostly cracks and delamination. These result in local changes of the stiffness of elements for such materials and consequently their dynamic characteristics are altered. This problem is well understood in case of constructing elements made of isotropic materials, while data concerning the influence of fatigue cracks on the dynamics of composite elements are scarce in the available literature. The research context is described in Sec.2.1. The focus of the thesis as well as the main objectives is discussed in Sec.2.2. The beam is manufactured from a Glass Fibre Reinforced Polymer (GFRP) and its box and Channel like beam. This beam was actually used as a prototype for footbridge. The GFRP (Glass Fibre Reinforced Polymer) composite materials are being utilized in more structures like bridges as the technology. GFRP composite are ideal for structural applications where high strength to weight and stiffness to weight ratios are needed. As the technology progresses, the cost involved in manufacturing and designing composite material will reduce, thus bringing added cost benefits also. The vibration analyses in composite beams have been a problem for structural designer for years and have increased recently. Though, all elements have natural frequencies with the potential to suffer excessive vibrations under dynamic load. This is done by using modal analysis, which allows one to determine the natural frequencies of the structure, associated mode shapes and damping. And once natural frequencies are known, thus making structure suitable for the task designed for. This is mainly due to the human feeling of vibration while crossing a footbridge with a frequency close to the first (fundamental) natural frequency of the bridge, although the vibration caused by the pedestrians are far from harmful to the bridge. Therefore vibration analysis of such structure can be considered to be a serviceability issue. Modal parameters of a structure are frequency, mode shape and damping. Frequency is directly proportional to structure's stiffness and inverse of mass. Nevertheless, modal parameters are functions of physical properties of the structure. Thus, changes in the physical properties such as, beginning of local cracks and/or loosening of a connection will cause detectable changes in the modal properties by reducing the structure's stiffness. The design of GFRP (Glass Fibre Reinforced Polymer) bridge deck established to promote the use of innovative material and lead to use in footbridge construction to improve the reliability for proper safety and serviceability. The Aberfeldy cable-stayed was the first GFRP footbridge, built back in 1992. There are only two GFRP footbridge existing in UK knows as the Halgavor and Willcot. The use of glass or carbon fibre reinforced polymer was due to its advantages, for they are easily drawn into having a high strength-to-weight ratio, low maintenance and lightweight.

II. GOVERNING EQUATION

The differential equation of the bending of a beam with a mid-plane symmetry (Bij = 0) so that there is no bending-stretching coupling and no transverse shear deformation (Ɛxz= 0) is given by;

$$IS_{11} \frac{d^4\omega}{dx^4} = q \cdot x$$

It can easily be shown that under these conditions if the beam involves only a one layer, isotropic material, then IS11= EI = Ebh3/12 and for a beam of rectangular cross-section Poisson‟s ratio effects are ignored in beam theory,

which is in the line with Vinson &Sierakowski (1991).
In Equation 1, it is seen that the imposed static load is written as a force per unit length. For dynamic loading, if Alembert"s Principle are used then one can add a term to Equation.1 equal to the product mass and acceleration per unit length. In that case Equation.1 becomes

$$IS_{11} \frac{d^4\omega}{dx^4} = q\,x,t - \rho A \frac{\partial^2 \omega(x,t)}{\partial x^2}$$

Where ω and q both become functions of time as well as space, and derivatives therefore become partial derivatives, ρ is the mass density of the beam material, and here A is the beam cross-sectional area. In the above, q(x, t) is now the spatially varying time-dependent forcing function causing the dynamic response, and could be anything from a harmonic oscillation to an intense one-time impact.
However, natural frequencies for the beam occur as functions of the material properties and the geometry and hence are not affected by the forcing functions; therefore, for this study let q(x, t) be zero.

III. MATHEMATICAL MODELING

The model chosen is a cantilever composite beam of uniform cross-section A, The width, length and height of the beam are B, L and H, respectively in Figure.3.1. The angle between the fibers and the axis of the beam is 'α'.

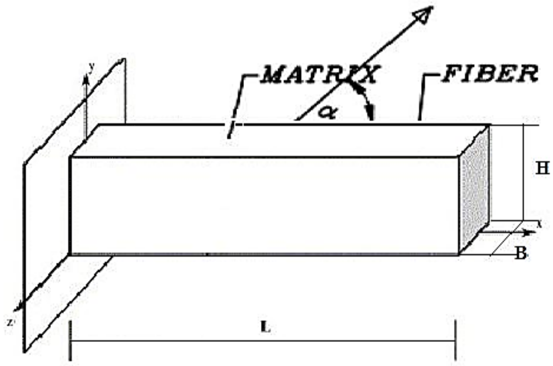

Figure 3.1 Schematic diagram cantilever composite beam

Vibration Study Analysis; Mass and stiffness matrices of each beam element are used to form global mass and stiffness matrices. The dynamic response of a beam for a conservative system can be formulated by means of Lagrange"s equation of motion in which the external forces are expressed in terms of time-dependent potentials and then performing the required operations the entire system leads to the governing matrix equation of motion

$$M\,q + K_e - P(t)\,K_g\; q = 0$$

where "q" is the vector of degree of freedoms. M, K_e, K_g are the mass, elastic stiffness and geometric stiffness matrices of the beam. The periodic axial force $P\;t = P_o + P_t Cos\Omega t$, where Ω is the disturbing frequency, the static and time dependent components of the load can be represented as a fraction of the fundamental static buckling load Pcr hence putting $P\;t = \alpha P_{cr} + \beta P_{cr}\;Cos\;\Omega t$. In this analysis, the computed static bucking load of the composite beam is considered the reference load. Further the above equation reduces to other problems as follows.
Free vibration with α = 0, β = 0 and ω = Ω/2 the natural frequency

$$K_e - \omega^2\, M\, q = 0$$

Static stability with α = 1, β = 0, Ω = 0

$$K_e - P_{cr}\;K_g\;q = 0$$

Element stiffness matrix;
Element stiffness matrix for a three-nodes composite beam element with three degrees of freedom δ = (u, v, θ) at each node, for the case of bending in the x, y plan, are given in the line Krawczuk&Ostachowicz (1995) as follows:

$$K_e = IS_{11v}[B]^T D\, B\, dv$$

$$\text{where } [B] = \frac{\partial}{\partial x^2} N = \text{strain displacement ma}$$

$$K_e = [K_{ij}]_{6\times 6} \text{ where } [K_{ij}]_{6\times 6} = (i = j = 1, 2..$$

$$k_{11} = k_{55} = 7BHS_{33}/3L_e,$$

$$k_{12} = k_{21} = -k_{56} = -k_{65} = BHS_{33}/2$$

$$k_{13} = k_{31} = k_{35} = k_{53} = -8BHS_{33}/3L$$

$$k_{14} = k_{41} = k_{36} = k_{63} = -k_{23} = -k_{32} = -k_{45} = -i$$

$$k_{15} = k_{51} = BHS_{33}/3L_e,$$

$$k_{16} = k_{61} = -k_{25} = -k_{52} = -BHS_{33}/6,$$

$$k_{22} = k_{66} = BH(7H^2 S_{11}/36L + L\,S_{33}/9)$$

$$k_{24} = k_{42} = k_{46} = k_{64} = BH(-2H^2 S_{11}/9L + L\,S_{33}/9)$$

$$k_{26} = k_{62} = BH(H^2 S_{11}/36L + L\,S_{33}/18)$$

$$k_{33} = -16BHS_{33}/3L_e,$$

$$k_{44} = BH(4H^2 S_{11}/9L + 4L\,S_{33}/9)$$

$$k_{34} = k_{43} = 0$$

where B is the width of the element, H is the height of the element and L denotes the length of the element. S_{11},S_{13} , and S_{33} are the stress-strain constants.

Generalized element mass matrix;
Element mass matrix of the non-cracked composite beam element is given in the line Krawczuk&Ostachowicz (1995) as

$$K_e=\rho_v[N]^T\ N\,dv$$

$$M_e=[M_{ij}]_{6\times6} \text{ where } [M_{ij}]_{6\times6}= (i=j=1, 2\ldots\ldots 6) \text{ are}$$

$$m_{11}= m_{55}=2\rho BHL_e/15,$$

$$m_{12}=m_{21}=-m_{56}=-m_{65}=\rho BHL^2\ /180,$$
$$m_{13}= m_{31}= m_{35}=m_{53}=\rho BHL_e/15,$$
$$m_{14}= m_{41}= -m_{45}=-m_{54}= -\rho BHL^2/ 90$$
$$m_{15}= m_{51}=-\rho BHL / 30$$
$$m_{16}= m_{61}= -m_{25}= -m_{52}=\rho BHL^2/180,$$
$$m_{22}= m_{66}=\rho BHL(L^2/1890- H^2/ 360),$$
$$m_{24}= m_{42}=m_{46}= m_{64}=\rho BHL(-L^2/ 945+ H^2/180),$$
$$m_{26}= m_{62}=\rho BHL(L^2/1890- H^2/ 360),$$
$$m_{33}=8\rho BHL /15,$$
$$m_{44}=\rho BHL(2L^2/ 945+2H^2/ 45),$$
$$m_{34}= m_{43}= m_{36}= m_{63}= m_{23}= m_{32}=0,$$

where ρ is the mass density of the element, B is the width of the element, H is the height of the element and L denotes the length of the element.

Bending test;The most commonly used test for ILSS is the short beam strength (SBS) test under three point bending. The SBS test was done as per ASTM D 2344/ D 2344 M (2006) by using the INSTRON 1195 material testing machine. The specimens were tested at 2, 50, 100, 200 and 500 mm/minute cross head velocities with a constant span of 34 mm to obtain interlaminar shear strength (ILSS) of samples. Before testing, the thickness and width of the specimens were measured accurately. The test specimen was placed on the test fixtures and aligned so that its midpoint was centered and its long axis was perpendicular to the loading nose. The load was applied to the specimen at a specified cross head velocity. Breaking load of the sample was recorded. About five samples were tested at each level of experiment and their average value along with standard deviation (SD) and coefficient of variation (CV) were reported in result part. The interlaminar shear strength was calculated using the formula,
S = (0.75Pb)/bd as per ASTM D 2344
Where Pb is the breaking load in kg; b is the width in mm and d is the thickness in mm.

Figure 4.1 (a): Three point bend test setup and fixture

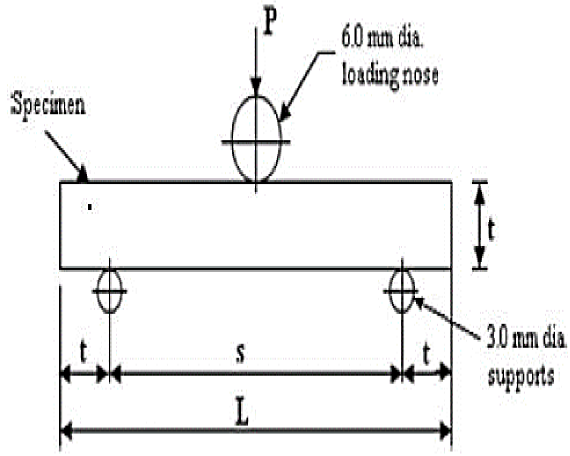

Figure 4.1 (b): Schematic diagram of three point bend test

Determination of material constants: Laminated composite plates behave like orthotropic lamina, the characteristics of which can be defined completely by four material constants i.e. E1, E2, G12, and V12 where the suffixes 1 and 2 indicate principal material directions. For material characterization of composites, laminate having eight layers was fabricated to evaluate the material constants. The constants are determined experimentally by performing unidirectional tensile tests on specimens cut in longitudinal and transverse directions, and at 45° to the longitudinal direction, as described in ASTM standard: D 3039/D 3039 M (2008). The tensile test specimens are having a constant rectangular cross section in all the cases. The dimensions of the specimen are mentioned below in Table 4.1.

Table 4.1: Size of the specimen for tensile test

Length(mm) Width(mm) Thickness(mm)	Width(mm)	Thickness(mm)
200	25	3

The specimens were cut from the plates themselves by diamond cutter or by hex saw as per requirement as shown in

Figure 4.2 (a). Four replicate sample specimens were tested and mean values were adopted. The test specimens are shown in Figure 4.2. (b) to Figure 4.2(d).

Figure 4.2: Diamond cutter for cutting specimens

Coupons were machined carefully to minimize any residual stresses after they were cut from the plate and the minor variations in dimensions of different specimens are carefully measured. For measuring the Young's modulus, the specimen was loaded in INSTRON 1195 universal testing machine (as shown in Figure 4.3) monotonically to failure with a recommended rate of extension (rate of loading) of 0.2 mm/minute. Specimens were fixed in the upper jaw first and then gripped in the movable jaw (lower jaw). Gripping of the specimen should be as much as possible to prevent the slippage and an extensometer respectively. Failure pattern of woven fiber glass/epoxy composite specimen is shown in Figure 4.4. From these data, engineering stress vs. strain curve was plotted; the initial slope of which gives the Young's modulus. The ratio of transverse to longitudinal strain directly gives the Poisson's ratio by using two strain gauges in longitudinal and transverse direction. But here Poisson.s ratio is taken as 0.3.
The values of material constants finally obtained experimentally for vibration are presented in Chapter-6.

Figure 4.3: Tensile test of woven fiber glass/epoxy composite specimens

Figure 4.4: Failure pattern of woven fiber glass/epoxy composite specimen

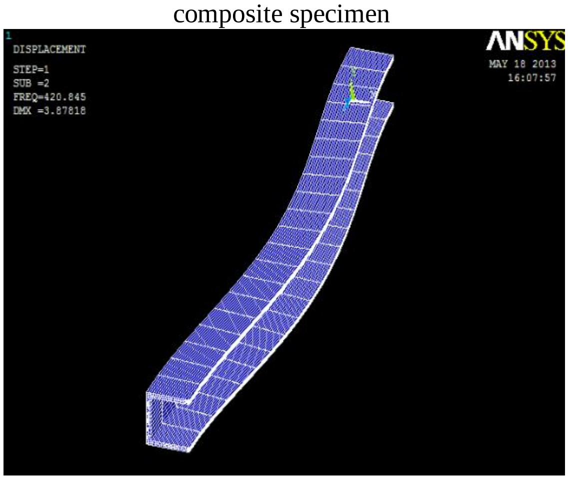

Figure 6.8: Modal analysis of a 8 layer channel beam at fixed-fixed boundary condition by Ansys

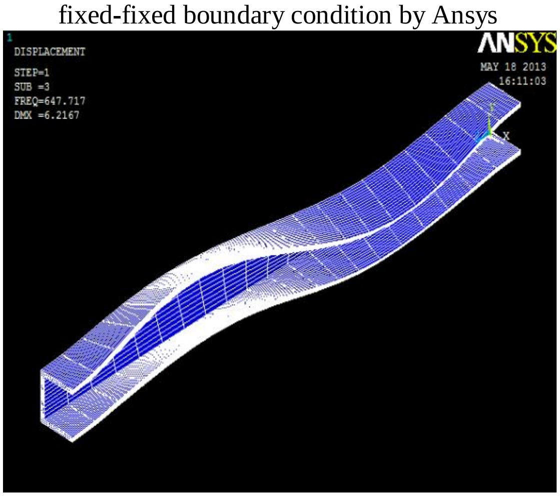

Figure 6.9: Four natural frequency mode shapes of a 8 layer channel beam at fixed-fixed boundary condition by Ansys

IV. CONCLUSION

The following conclusions can be made from the present investigations of the box and channel shaped composite beam finite element. This element is versatile and can be used for static and dynamic analysis of a composite or isotropic beam. The natural frequencies of different boundary conditions of composite beam have been reported.

The program result shows in general a good agreement with the existing literature. It is found that natural frequency is minimum for clamped –free supported beam and maximum for clamped-clamped supported beam. Mode shape was plotted for differently supported laminated beam with the help of ANSYS [58] to get exact idea of mode shape. Vibration analysis of laminated composite beam was also done on ANSYS [58] to get natural frequency and same trend of natural frequency was found to be repeated. There is a good agreement between the experimental and numerical results. The Finite Element method defined previously is directly applied to the explained examples of generally laminated composite beams to obtain the natural frequencies, the impact of Poisson effect, slender ratio, material anisotropy, shear deformation and boundary conditions on the natural frequencies of the laminated beams are analyzed. And it is found that the present results are in very good agreement with the theoretical results of references. We assumed different examples and it is found that natural frequencies increase with the value of E1 increases. It is found that natural frequencies decrease with the increase of beam length. It is observed that natural frequency increases with increase in number of layers and aspect ratios for both box and channel shaped beams. The material anisotropy has a relatively negligible effect on the mode shapes and the slenderness ratio has considerable effect on all five modes especially on the fifth mode.

Scope of future work: An analytical formulation can be derived for modelling the behaviour of laminated composite beams with integrated piezoelectric sensor and actuator. Analytical solution for active vibration control and suppression of smart laminated composite beams can be found. The governing equation should be based on the first-order shear deformation theory. The dynamic response of an unsymmetrical orthotropic laminated composite beam, subjected to moving loads, can be derived. The study should be including the effects of transverse shear deformation, rotary and higher-order inertia. And also we can provide more number of degree of freedom about 10 to 20 and then should be analyzed by higher order shear deformation theory. The free vibration characteristics of laminated composite cylindrical and spherical shells can be analyzed by the first-order shear deformation theory and a meshless global collocation method based on thin plate spline radial basis function. An algorithm based on the finite element method (FEM) can be developed to study the dynamic response of composite laminated beams subjected to the moving oscillator. The first order shear deformation theory (FSDT) should be assumed for the beam model. The damping behavior of laminated sandwich composite beam inserted with a visco elastic layer can be derived.

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