FINITE ELEMENT ANALYSIS OVER MULTIPLE CRACKED UNIFORM AND VENTURED BEAMS TO RECOVER FROM VIBRATION

Gaurav Kumar[1], Prof. Dharmendra Singh[2]
[1]Scholar M.Tech (Structure) Department of Civil Engineering, RNTU, Bhopal (M.P).
[2]Guide, Department of Civil Engineering, RNTU, Bhopal (M.P).

ABSTRACT: In this paper the crack considered is transverse open crack it has been analysed that when the crack is present in beam the reduced stiffness matrix can be found using Fracture mechanics theory. Due to the importance of this problem, the FEM formulation is done for cracked uniform and stepped beams. Analysis includes free vibration analysis of the Cantilever Bernoulli-Euler beam of various cross-sections. The effects of various parameters such as natural frequencies for uniform and stepped beams with and without cracks are presented and convergence study is done. Comparisons of the natural frequencies of the beams with the pervious papers in order to understand the accuracy of present study is included. Numerical Analysis is done considering an Aluminium beam (cantilever beam) with transverse open crack in order to obtain the natural frequencies of uniform beam and stepped beams with out and with multiple cracks. The results are obtained by using Finite Element Method (FEM) in MATLAB environment to find out the overall stiffness matrix, natural frequencies and non- dimensional frequencies.

I. INTRODUCTION

Engineering structures are designed to withstand the loads they are expected to be subject to while in service. Among them Beams are a standout amongst the most usually utilized structural components within various structural elements in numerous engineering applications and experience a wide mixed bag of static and element loads. Beams are widely used as structural components in engineering applications and also provide a fundamental model for many engineering applications. Aircraft wings, helicopter rotor blades, spacecraft antennae, and robot arms are all examples of structures that may be modeled with beam-like elements. Beam sort structures are being generally utilized in steel shaped structure and manufacturing of machines.

Beams with variable cross-section and/or material properties are frequently used in aeronautical engineering (e.g., rotor shafts and functionally graded beams), mechanical engineering (e.g., robot arms and crane booms), and civil engineering (e.g., beams, columns, and steel composite floor slabs in the single direction loading case). Stepped beam-like structures are widely used in various engineering fields, such as robot arm and tall building, etc.

II. RESEARCH SIGNIFICANCE

In numerous engineering applications beams are universally used structural elements which experience a wide variety of static and dynamic loads. During their utilisation various engineering structures subjected to degenerative effects, all these are responsible for the development of cracks. The propagation of these cracks decreases the stiffness of an element and sometimes leads to the failure of the complete structure. Immediate detection of these cracks is an important task of an engineer to determine the effect of crack on stiffness on the beam, all these beams or shafts subjected to these conditions are modelled using either Timoshenko beam or Euler-Bernoulli theories. The characteristic equation involving natural frequency, the crack depth and crack location and other properties of the beam are derived using conventional methods like boundary conditions of the beam along with the stress intensity factors. The change in dynamic characteristics of multiple cracked stepped beams with varying cross sections using FEM. This problem has been a subject of many papers, but only a few papers have been devoted to the changes in the dynamic characteristics of multiple cracked stepped beams with varying cross sections using FEM.

III. METHODOLOGY

Mathematical Formulation for uniform beam of rectangular cross-section:

Considering a typical cracked uniform beam element of rectangular cross-section of breadth 'b', depth 'h' with a depth of crack 'a'. The left hand side end node '***i***' is assumed to be fixed, while the right hand side end node '***j***' is subjected to shearing force P_1 and bending moment P_2. The corresponding generalized displacements are denoted as q_1 and q_2 as shown in Figure 3.2. The equation governed of the vibrated analysis of the uniform beam along an open transverse crack are computed on basis of the model proposed by Zheng D. Y. and Kessissoglou N. J. K. (2004).

L_C = Distance between the right hand side end node j and the crack location.

Le = Length of the beam element.

A = Cross-sectional area of the beam.

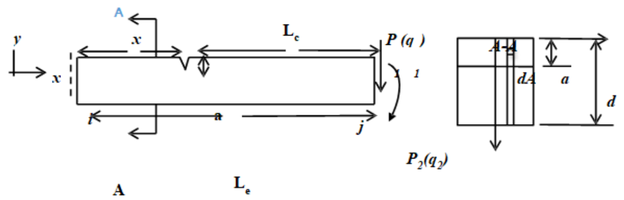


Figure : A typical cracked beam element subjected to shearing force and bending moment of rectangular cross-section

The governing equations for the vibration analysis of the uniform beam with an open transverse crack are followed as following

According to Zheng [2004], the additional strain energy due to existence of crack can be expressed as

$$\pi = \int_{A} G\, dA_c$$

Where, G = the strain energy release rate and

AC = the effective cracked area

$$G = \frac{1}{E}\left[\left(\sum_{n=1}^{2} K_{In}\right)^2 + \left(\sum^{2} K\right)^2 + \left(\sum^{2} K\right)^2\right]$$

Mathematical Formulation for Uniform Beam of Circular Cross-section:

Considering a typical cracked uniform beam element of circular cross-section of diameter 'D' with a crack of depth 'a'. The left hand side end node 'i' is assumed to be fixed, while the right hand side end node 'j' is subjected to axial force P_1, shearing force P_2 and bending moment P_3 as shown in Figure 3.3.

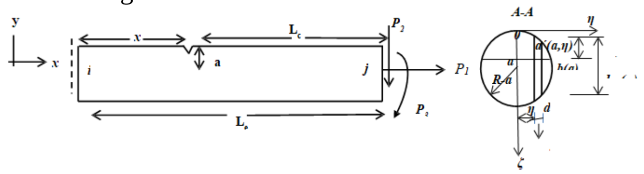


Figure 3.3: A typical cracked beam element subjected to shearing force and bending moment of circular cross-section.

The equations governed for the vibration analysis of uniform beam along an open transverse crack are computed on basis of the model proposed by Zheng D. Y. and Kessissoglou N. J. K. (2004).The geometrical dimensions are as follows:

$$\xi' = \xi + \sqrt{\frac{D^2}{4} - \eta^2} - \frac{D}{2}$$

$$b(a) = \sqrt{Da - a^2}$$

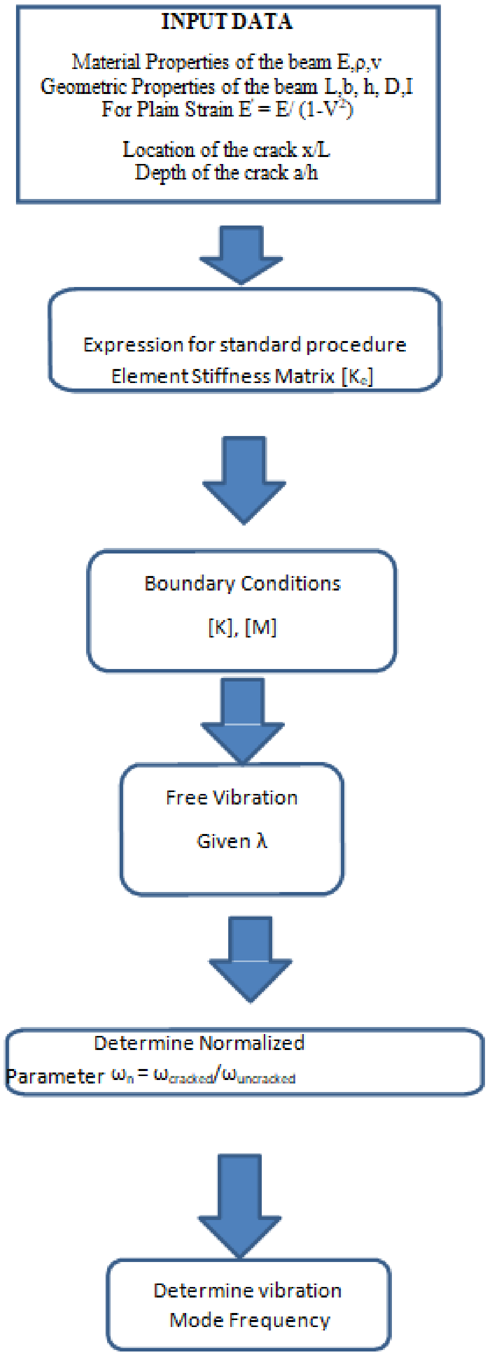
$$h'(\eta) = \sqrt{D^2 - 4\eta^2}$$

$$a'(a,\eta) = \sqrt{\frac{D^2}{4} - \eta^2} - \left(\frac{D}{2} - a\right)$$

where D is the diameter of the beam. A similar procedure to the rectangular cross-sectional beam is used to derive the overall additional flexibility matrix for a circular cross-sectional beam. The additional strain energy due to the existence of the crack can be expressed as

$$\Pi_c = \int_{A_c} G dA = \int_{-b(a)}^{b(a)} \left[\int_{0}^{a'(a,\eta)} G d\xi' \right]$$

FLOW CHART



IV. RESULTS AND DISCUSSION

This part contains

- Verification with past studies

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- Results of numeric.

Comparison with Previous Studies

In order to check the accuracy of present analysis and to understand the results of the free vibration of the Bernoulli-Euler beam ,the effect of various parameters with multiple cracks are presented .The natural frequencies of the beams are compared with the pervious papers.

This includes

- Comparison of analysis of freely vibrated beam of uniform and even stepped beam of rectangular cross-sections with multiple cracks.
- Comparison of Free Vibrational analysis of uniform beams of circular cross-sections with multiple cracks.

Free Vibration Analysis of Cracked Uniform Cantilever Beam

Case (1):- Comparison of natural frequencies for a cantilever beam with single crack with results of Shiffrin(1999)

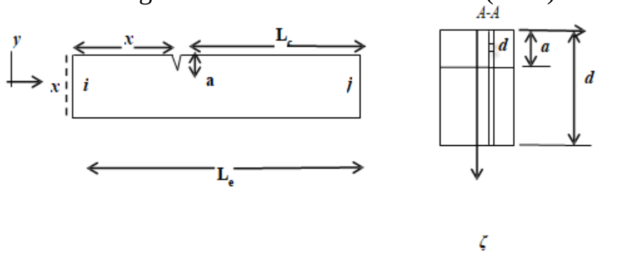

Figure 4.4: Sketch of Cantilever beam with single crack of rectangular cross-section

Table 1: Comparison of natural frequency of single cracked uniform cantilever beam with F.E.M

Elastic modulus of the beam = 210MPa, Poisson's Ratio = 0.3, Density = 7800 kg/m^3, Beam Width = 0.02m, Beam depth = 0.02 m, Beam length = 0.8m, Position of the crack from clamped end x_1= 0.12m, Crack depth a_1=0.002 m.

Modes	Natural frequency (Hz) Shiffrin	Present analysis using FEM	% Error
Mode 1	26.1231	26.1673	0.168
Mode 2	164.0921	164.1292	0.022
Mode 3	459. 6028	459.620	0.004

Case (2):- Comparison of natural frequencies for a cantilever beam with double crack with results of Shiffrin(1999)

Table 2: Comparison of natural frequency of doubled cracked uniform cantilever beam with F.E.M

Position and crack depth of first crack: a_1=0.002m; x_1=0.12 m Position and crack depth of second crack: a_2=0.003m; x_2=0.4m

Mode Number	Natural frequency (Hz)Shiffrin	Present analysis using FEM	% Error
1	26.0954	26.1539	0.223
2	164.3221	164.7585	0.266
3	459.6011	459.6173	0.003

From Table 1 and Table 2 it is, observed that natural frequencies of Shiffrin(1999) agrees with the present MATLAB analysis using FEM formulation in case of both single and double cracks.

Case (3):- Comparison of uniform cantilever beam of square cross-section with multiple cracks with results of Mostafa Attar(2012)

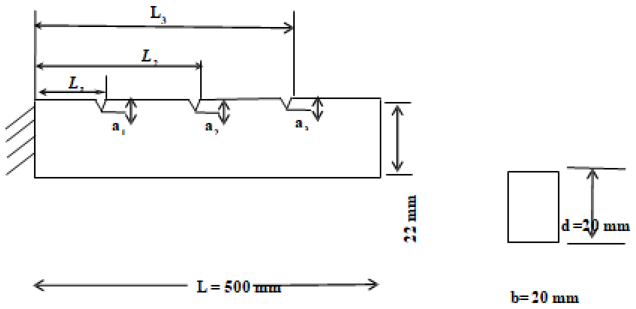

Figure 4.5: Sketch of Triple cracked uniform rectangular beam

Table 3: Comparison of Natural frequency of triple cracked cantilever beam with TMM and FEM

Elastic modulus of the beam = 210MPa, Poisson's Ratio = 0.3, Density = 7860 kg/m3, Beam width = 0.02m, Beam depth = 0.02 m, Beam length = 0.5m

Case	Crack Location			Methods	Natural Frequencies ω_s (rad/s)					
	x_1	x_2	x_3		ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
1	0.2	0.4	0.6	Present study	418.8631	2624.823	7352.854	14411.38	23789.01	35655.29
				FEM*[19]	416.8933	2612.065	7323.879	14356.68	23589.91	35603.94
				% Error FEM	0.470	0.486	0.394	0.379	0.836	0.144
				TMM*[19]	416.9159	2612.213	7324.210	14357.28	23592.02	35604.06
				% Error TMM	0.464	0.480	0.389	0.375	0.828	0.143
2	0.2	0.4	0.8	Present study	418.9152	2627.313	7351.134	14394.25	23790.05	35646.15
				FEM*	417.0652	2620.375	7318.436	14299.97	23600.29	35573.62
				% Error FEM	0.441	0.264	0.444	0.654	0.797	0.203
				TMM*	417.0864	2620.455	7318.811	14301.02	23602.31	35574.00
				% Error TMM	0.436	0.261	0.439	0.647	0.789	0.202
3	0.2	0.6	0.8	Present study	419.0779	2626.284	7350.398	14392.93	23799.17	35654.07
				FEM*	417.6291	2617.683	7315.436	14300.48	23601.47	35573.74
				% Error FEM	0.345	0.327	0.475	0.642	0.830	0.225
				TMM*	417.6464	2617.786	7315.833	14301.53	23603.48	35574.12
				% Error TMM	0.341	0.323	0.470	0.635	0.822	0.224
4	0.4	0.6	0.8	Present study	419.4202	2624.264	7348.95	14405.69	23784.43	35653.36
				FEM*	418.7431	2610.199	7311.806	14337.70	23575.09	35597.99
				% Error FEM	0.161	0.535	0.505	0.471	0.880	0.155
				TMM*	418.7517	2610.361	7311.243	14338.46	23577.32	35598.16
				% Error TMM	0.159	0.529	0.513	0.466	0.870	45 0.154

In Table 3 it is observed that natural frequencies of Mostafa Attar (2012) agrees with the present MATLAB analysis using FEM formulation for all modes but in mode 5 we observe more percentage error in case of both TMM (Transfer Matrix Method) and FEM (Finite Element Method) compared to other modes.

Free Vibration Analysis of Uncracked Stepped Beams

In this solution it associates the computation of frequencies occurred naturally for Uncracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by using methods Discrete Singular convolution (DSC), Differential Quadrature Element method (DQEM), Finite Element Method (FEM), Composite Element Method (CEM) given by Guohui D and Xinwei W(2013). Elastic modulus = 71.7GPa, Density = 2830 kg/m3, Width b = 20mm, Depth of the beam h_1= 19.05 mm, Depth of the stepped beam h_2= 5.49 mm, Length of the beam = 254 mm, Length of the stepped beam = 140 mm

Case of- Comparison of Natural Frequencies of Twelve Stepped Cantilever Beam

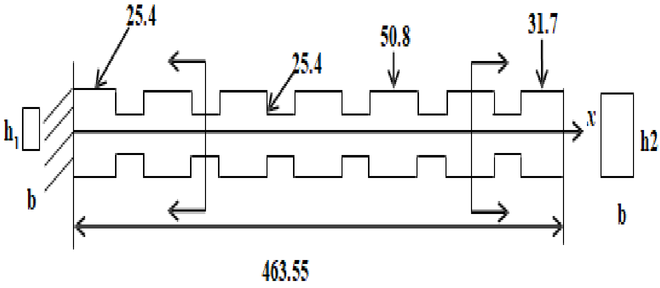

Figure 4.8: Sketch of a twelve stepped Cantilever beam

Table 6: Comparison of natural frequencies of a twelve stepped cantilever beam

Elastic modulus = 60.6GPa, Density = 2664 kg/m^3, Width b = 3.175 mm, Depth of the beam h_1=
12.7 mm, Depth of the stepped beam h_2= 25.4 mm, Total length of the beam = 463.55 mm

DSC	DQEM	CEM	% Error DSC, DQEM	% Error CEM	FEM	Experiment	% Error Experiment	Present analysis using FEM
1	54.496	54.695	0.0009	0.002	54.795	54.985	0.004	54.496
2	344.793	344.808	0.0052	0.0008	-	344.807	0.001	344.811
4	977.740	977.812	0.0170	0.009	-	977.809	0.009	977.906
5	1951.199	1951.409	0.0482	0.037	-	1951.398	0.038	1952.14
10	3301.141	3301.639	0.1098	0.094	-	3301.606	0.095	3304.77

From Table 6 it is observed that natural frequencies of Guohui D and Xinwei W(2013) agrees with the present MATLAB analysis using FEM formulation with less percentage error in case of FEM, DQEM , DSC and CEM.

Free Vibration Analysis of Cracked Stepped Beams of Rectangular Cross-Section

In this solution it associates the computation of frequencies occurred naturally for cracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by Ameneh M (2012) using a novel local flexibility-based damage index method.

Case (1):- Comparison of Single Cracked Two Step Cantilever Beam

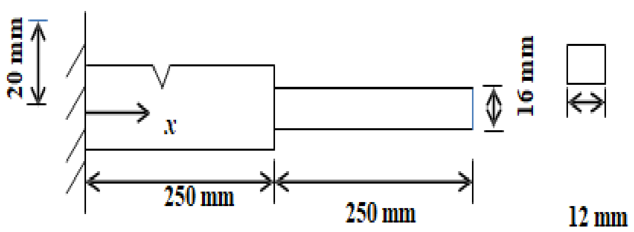

Figure 4.9: Sketch of Single Cracked Two Step Cantilever Beam

Table 7: Comparison of Single Cracked Two Step Cantilever Beam with Novel local Flexibility-based damage index method

Elastic modulus = 210MPa, Density = 7800 kg/m^3, Poisson's ratio =0.3, Width b = 12 mm, Depth of the beam h_1= 20 mm, Depth of the stepped beam h_2= 16 mm, Total length of the beam = 500mm

Case No	Location of Crack	Crack Depth	Natural Frequencies (Hz)			
			ω_1	ω_2	ω_3	ω_4
1	0.05	0.2	71.492	372.24	1041.6	2007.8
Reference[1]			*70.514	366.64	1024.5	1950.4
% Error			1.367	1.504	1.641	2.858
2	0.1	0.5	60.759	343.55	1033.3	2026.8
Reference			*61.379	353.62	1024.1	1925.2
% Error			1.010	2.847	0.890	5.012
3	0.2	0.4	66.456	375.59	1033.0	1921.8
Reference			* 67.529	371.85	1019.2	1875.0
% Error			1.588	0.995	1.335	2.435
4	0.3	0.3	71.185	375.96	1030.1	2003.5
Reference			*70.652	371.94	1005.3	1932.4
% Error			0.748	1.069	2.407	3.548
5	0.4	0.5	63.738	339.40	926.24	2017.6
Reference			*68.711	355.75	968.21	1955.2
% Error			7.237	4.595	4.334	3.092
6	0.45	0.1	72.477	376.53	1053.8	2027.4
Reference			*72.334	372.12	1034.6	1959.9
% Error			0.197	1.171	1.821	3.329
7	0.55	0.1	72.4827	376.33	1054.2	2026.0
Reference			*72.385	371.55	1036.5	1957.0
% Error			0.134	1.270	1.678	3.405
8	0.6	0.3	72.183	367.23	1047.8	1167..9
Reference			*71.996	360.21	1027.7	1039.3
% Error			0.259	1.911	1.918	11.011
9	0.7	0.2	72.472	375.11	1047.3	2025.
Reference			*72.370	369.55	1022.6	1957.5
% Error			0.140	1.482	2.358	3.333
10	0.8	0.4	72.4198	369.88	992.57	1880.9
			*72.383	367.76	989.29	1849.4
% Error			0.050	0.573	0.330	1.674
11	0.2	0.06	72.478	376.8	1054.1	2027.7
Reference			*72.310	372.94	1035.5	1960.2
% Error			0.231	1.024	1.764	3.328
12	0.65	0.085	72.498	376.56	1053.7	2028.2
Reference			*72.426	372.24	1034.4	1963.5
% Error			0.099	1.147	1.831	3.190

In Table 7, the first six cases where crack is present in the first half of the beam and the second six cases are where crack is present in step of the beam we observe that natural frequencies of Ameneh M (2012) agrees with the present MATLAB analysis using FEM formulation. Case 5 and Case 8 show quite high percentage error.

Case (2):- Comparison of Double Cracked Two Step Cantilever Beam

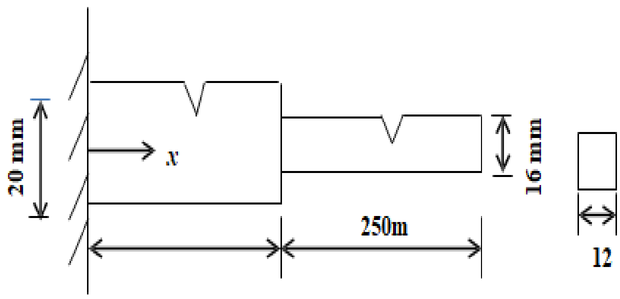

Figure 4.10: Sketch of Double Cracked Two Step Cantilever Beam

Table 8: Comparison of Double Cracked Two Step Cantilever Beam with Novel local Flexibility-based damage index method

Case No	Simulated data				Natural frequencies (Hz)			
	Crack No.1		Crack No.2					
	β	γ	β	γ	ω_1	ω_2	ω_3	ω_4
1	0.1	0.2	0.4	0.2	71.489	373.966	1047.92	1027.55
Ref[1]					* 70.349	367.24	1023.9	1962.0
% Error					1.594	1.798	2.292	3.232
2	0.25	0.1	0.35	0.3	71.385	374.457	1029.71	2021.28
Ref					* 70.762	369.96	1003.5	1950.9
% Error					0.872	1.200	2.545	3.481
3	0.45	0.3	0.2	0.2	69.973	370.669	1038.89	1971.72
Ref					* 70.474	365.46	1021.4	1919.3
% Error					0.710	1.405	1.683	2.658
4	0.55	0.3	0.7	0.4	71.486	339.83	931.50	1858.92
Ref					* 70.918	336.9	980.7	1835.5
% Error					0.795	0.862	5.016	1.259
5	0.6	0.5	0.8	0.4	68.314	286.64	925.93	1623.97
Ref					* 70.833	329.65	972.81	1750.5
% Error					3.556	13.047	4.819	7.228
6	0.75	0.4	0.6	0.3	71.99	355.30	973.71	1896.36
Ref					* 72.024	355.39	974.8	1863.6
% Error					0.047	0.025	0.111	1.727
7	0.1	0.2	0.7	0.3	71.598	369.139	1029.146	2018.126
Ref					* 70.598	361.66	1002.9	1949.5
% Error					1.396	2.026	2.550	3.400
8	0.3	0.2	0.6	0.5	68.28	295.86	1000.207	1890.88
Ref					* 70.148	333.62	995.9	1880.2
% Error					2.662	11.318	0.430	0.564

From Table 8 it is observed that natural frequencies of Ameneh.M (2012) agree with the present MATLAB analysis using FEM formulation in case of double cracks. In Case 5 and Case 6 Show quite high percentage error for Mode 2.

Case (3):- Comparison of Triple Cracked Two Step Cantilever Beam

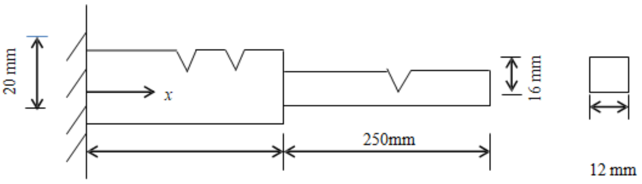

Figure 4.11: Sketch of Triple Cracked Two Step Cantilever Beam

Table 9: Comparison of Triple Cracked Two Step Cantilever Beam with Novel local flexibility-based damage index method.

Case no.	Crack no.1		.2		Crack no.3		Natural Frequencies (Hz)		Natural Frequencies (Hz)	
	Crack Location	Crack Depth	Crack Location	Crack Depth	Crack Location	Crack Depth	ω_1	ω_2		ω_3
1	0.15	0.3	0.3	0.45	0.7	0.35	63.272	359.59		904.08
Ref							*65.272	356.98		927.95
%Error							3.064	0.725		2.572
2	0.1	0.15	0.35	0.25	0.6	0.35	71.014	358.22		1028.6
Ref							*70.025	352.22		1011.1
%Error							1.392	1.674		1.701

From Table 9, it is observed that natural frequencies of Ameneh M,(2012) agrees with the present MATLAB analysis using FEM formulation in case of both triple cracks. In case 2 we observe quite high percentage error for 6th mode.

Free Vibration Analysis of Uniform Simply Supported Shaft

Case (1):- Comparison of Natural Frequencies of Fixed-Free Circular beam without crack

The problem involves calculation of natural frequencies for un-cracked Bernoulli-Euler Cantilever beam. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by Zheng D. Y (2004) using Finite Element Method using Gauss quadrature.

Elastic modulus = 206 GPa, Density = 7800 kg/m^3, Poisson's ratio =0.3, Diameter of the beam D = 0.03 m, Total length of the beam L= 1.0 m

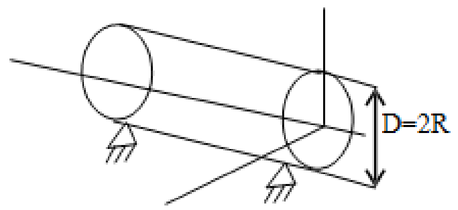

Figure 4.12: Sketch of Uniform Simply supported shaft

Table 10: Comparison of natural frequencies of uniform simply supported shaft

Mode	Natural Frequency (Hz) Present Analysis	Natural Frequency (Hz) (D.Y Zheng)	%Error
Mode1	60.539	60.543	0.006
Mode2	242.204	242.177	0.011
Mode3	544.941	544.936	0.0009

Analysis of freely vibrated Cracked Beams of uniform with Circular Cross-Section Case (1):- Free Vibration Analysis of Uniform Simply supported shaft with single crack

In this solution it associates the computation of frequencies occurred naturally for cracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained by Zheng D. Y (2004) using Finite Element Method using Gauss quadrature.

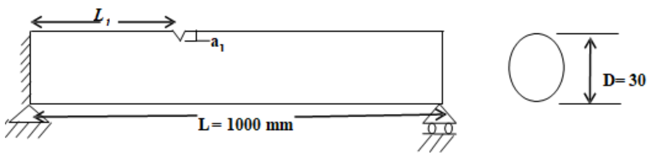

Figure 4.13: Sketch of Single cracked uniform Simply-Supported beam of circular cross-section

Table 11: Comparison of natural frequencies of single cracked uniform cantilever beam of circular cross-section

Mode	Natural Frequency (Hz) Present Analysis	Natural Frequency (Hz) (D.Y Zheng)	%Error
Mode1	56.008	55.92	0.157
Mode2	242.16	242.18	0.008
Mode3	506.90	506.85	0.009

Table 11 shows the percentage error graph, we observe that natural frequencies of Zheng D.Y(2004) agrees with the present MATLAB analysis using FEM formulation in case of both without and with crack.

Case (2):- Single Cracked Uniform Beam of Circular cross-section

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In this solution it associates the computation of frequencies occurred naturally for cracked Bernoulli-Euler beam of cantilever type. The results calculated using Finite Element Analysis in MATLAB and are validated with the results obtained using numerical model by Kisa.M (2006) which adds with finite elemental and structure synthesis mode procedure for analysis of beams with cross section of circular.

Material Parameters: Elastic modulus = 216 GPa Density = 7850 kg/m^3 Poisson ratio =0.33

Geometric Parameters:

Total length of the beam = 2.0 m

Here three different diameters are considered 1. R/L=0.1 (D=0.2L)

2. R/L=0.06 (D=0.12L)
3. R/L=0.04 (D=0.16L)

For all the above three cases the crack is located at $L_1/L=0.2$

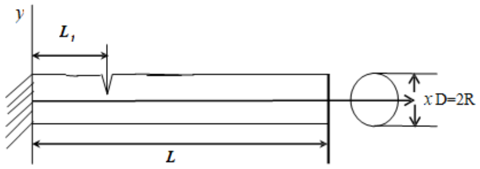


Figure 4.14: Sketch of Single cracked cantilever beam of circular cross-sectionCase (3):- Multi-cracked Uniform Circular Cantilever Beam Material Parameters:

Elastic modulus = 216 GPa Density = 7850 kg/m^3 Poisson ratio =0.33 Geometric Parameters:

Total length of the beam = 2.0 m R/L ratio = 0.04 (D=0.08L)

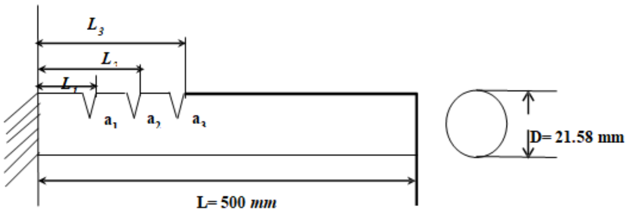


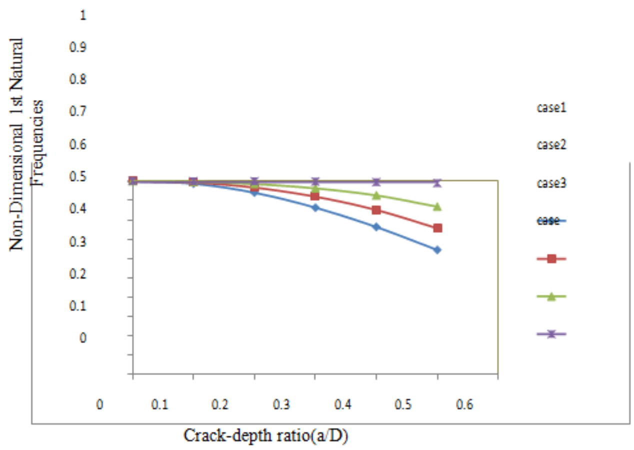
Figure 4.16: Sketch of multi-cracked beam of circular cross-section

Case 1 = $L_1/L=0.1$, $L_2/L=0.2$, $L_3/L=0.3$

Case 2 = $L_1/L=0.1$, $L_2/L=0.5$, $L_3/L=0.9$

Case 3 = $L_1/L=0.4$, $L_2/L=0.7$, $L_3/L=0.6$

Case 4 = $L_1/L=0.7$, $L_2/L=0.8$, $L_3/L=0.9$



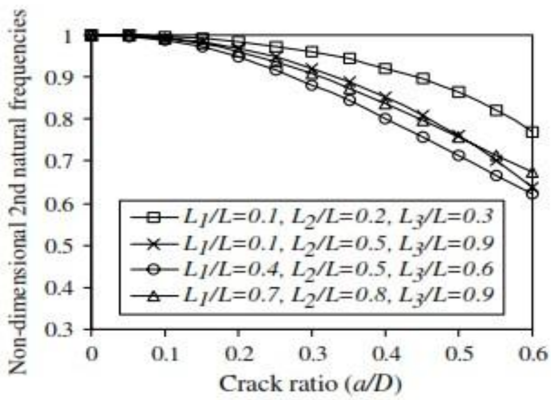
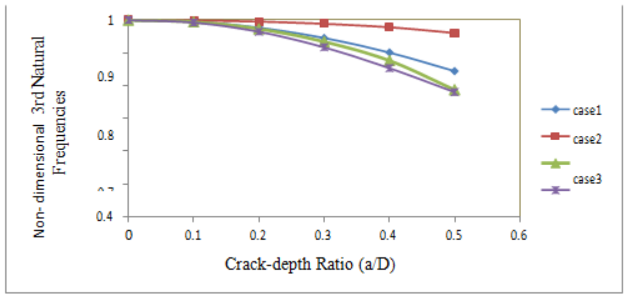


Figure 4.18: Comparison of 2nd non- dimensional natural frequencies of Triple cracked beam of circular cross-section with Component mode Synthesis Method



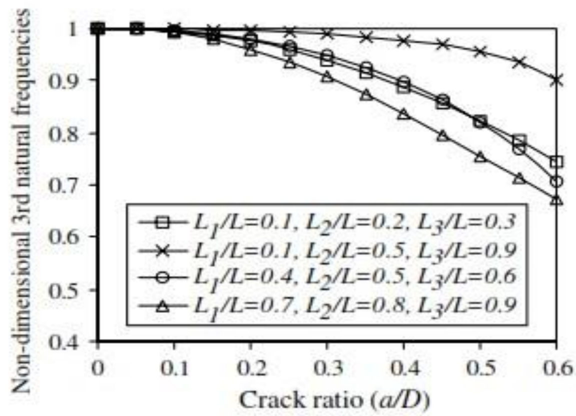


Figure 4.19: Comparison of 3rd non- dimensional natural frequencies of Triple cracked beam of circular cross-section with Component mode Synthesis Method

From Figures 4.17-4.19 it is observed that irrespective of single and multiple cracks of cantilever beam of circular cross-section the comparison of non- dimensional natural frequencies of present analysis using FEM agrees with Kisa.M (2006) which used FEM and Component mode Synthesis Method.

Numerical Results

Analysis of vibration subjected freely of the Euler-Bernoulli beam of multiple fractures considering the effect by various parameters such as crack location, crack depth ratio, numbers of cracks are presented. The method described has been used to analyse uniform and stepped beams considering Aluminum as the material property of the beam. The Normalized frequencies are found as ratio of frequency occurred naturally for a fractured beam / frequency occurred naturally of the un-fractured beam. The results are obtained by implementing the methodology given in Chapter 3 using Finite Element Method (FEM) in the MATLAB environment.

Following types of beams have been considered for the analysis Material Properties:
Elastic modulus of the beam = 70 GPa Poisson's Ratio = 0.35
Density = 2700 kg/m

Uniform Beam with Multiple Cracks

- Uniform beam of rectangular and circular cross-sections with multiple cracks.

Stepped Beam with Multiple Cracks

- Uniform beam, Single step beam and Two step beam with multiple cracks of rectangular cross-sections.

The results are analysed in the following manner

- Comparison between uniform beam of rectangular and circular cross-sections without crack and single crack and multiple cracks at respective locations.
- Effect of single step and two steps present in beams of rectangular cross-section without crack and with single crack are compared.
- Variation of frequencies with respect to single, double, multiple cracks in two step cantilever beam.

4.4.1 Uniform Beam with Multiple Cracks
Case (1):- Comparison between Uniform Beam of Rectangular and Circular cross-sections without Crack
Dimensions of the rectangular beam:
Beam width = 0.12m Beam depth = 0.22 m Beam length = 0.5 m

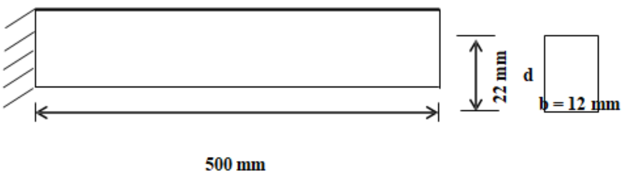


Figure 4.20: Sketch of uniform cantilever beam of rectangular cross-section

Dimensions of the circular beam:
Diameter of the beam D = 0.02158 m Length of the beam = 0.5 m

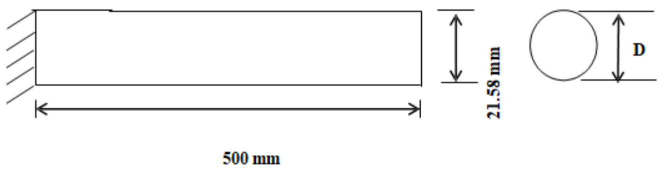


Figure 4.21: Sketch of uniform cantilever beam of circular cross-section

Mode	Natural Frequency of Rectangular beam (Hz)	Natural Frequency of Circular beam (Hz)	% Error
1	59.221	61.489	3.688
2	371.135	385.338	3.685
3	1039.196	1078.964	3.685
4	2036.448	2114.38	3.685

Table 12: Comparison of natural frequencies of uniform beam of rectangular cross-section with circular cross-section

Moment of inertia of the rectangular beam and circular beam is considered equal and comparison is done. From Table 12, it is observed that the % error is almost same for all the modes for uniform rectangular beam and uniform circular beam.

Case (2):- Comparison between Uniform Beam of Rectangular and Circular cross-sections with Single Crack
Case (a):- Location of Single Crack

- Case R1 = $L_1/L=0.1$
- Case R2 = $L_1/L=0.5$
- Case R3 = $L_1/L=0.85$

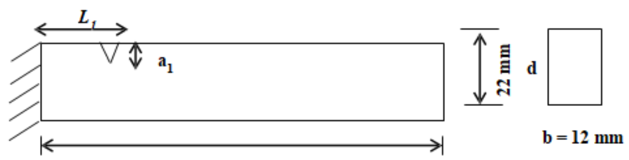


Figure 4.22: Sketch of uniform cantilever beam with single crack of rectangular cross-section

Case (b):- Location of Single Crack

- Case C1 = $L_1/L=0.1$
- Case C2 = $L_1/L=0.5$
- Case C3 = $L_1/L=0.85$

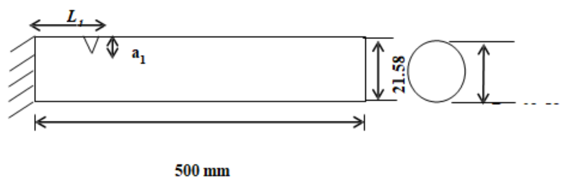


Figure 4.23: Sketch of uniform cantilever beam with single crack of circular cross-section

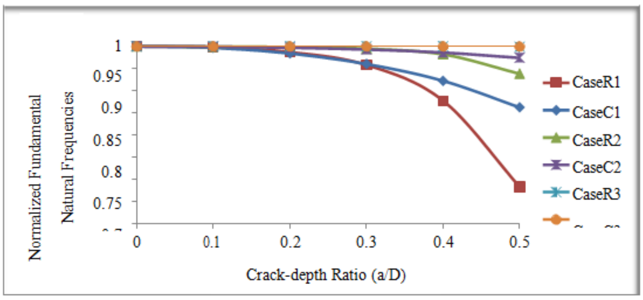


Figure 4.24: Comparison of Normalized fundamental natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Figure4.24, it is observed that in case of rectangular cross-section the normalized fundamental natural frequencies reduction is high than that of circular cross-section for all the cases respectively. It also shows that when crack is located near the fixed end of the beam the fundamental natural frequencies reduction is

higher and when crack is positioned at free end of beam the fundamental natural frequencies are generally unharmed although when the depth of crack is quite more When crack-depth ratio is 0.5 the natural frequency reduction of rectangular
beam is 25% more than circular beam when crack is located near to fixed end of the beam.

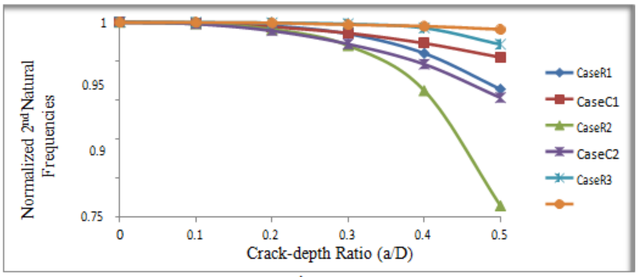


Figure 4.25: Comparison of normalized 2nd natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

In Figure4.25 it is observed that in case of rectangular cross-section the normalized 2nd natural frequencies difference is high than that of circular cross-section for all the cases respectively. The Figure shows when crack is located at center of the beam the 2nd natural frequencies reduction is higher and when crack-depth ratio is 0.5 the frequency reduction for rectangular beam in more by 15% than circular beam. It is also observed that when crack is located near free end of the circular beam the 2nd natural frequencies are generally unharmed although when the depth of crack is quite more.

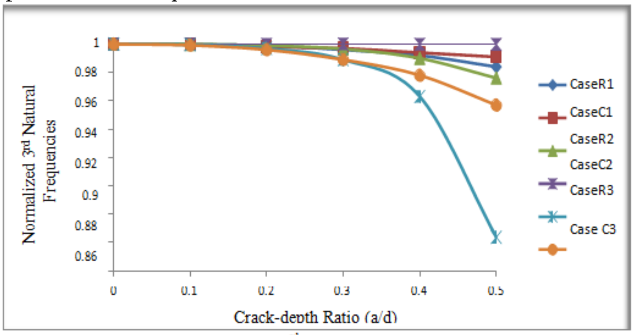


Figure 4.26: Comparison of Normalized 3rd natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Figure4.26, it is observed that in case of rectangular cross-section the Normalized 3rd natural frequencies difference is high than that of circular cross-section for all the cases respectively. It also shows that when crack is located at the free end of the beam the 3rd natural frequencies reduction is higher and for crack-depth ratio 0.5 the frequency reduction is more by 10% for rectangular beam than circular beam. When crack is located near fixed end of the circular beam the 3rd natural frequencies are generally unharmed although when the depth of crack is quite more.

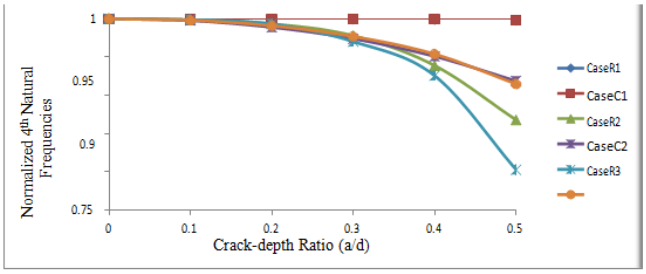


Figure 4.27: Comparison of Normalized 4th natural frequencies of single cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio

In Figure4.27 it is observed that in case of rectangular cross-section the Normalized 4th natural frequencies difference is high than that of circular cross-section for all the cases respectively. It also shows that when crack is located at center and end of the rectangular beam the 4th natural frequencies reduction is higher, it is observed that when crack is located at center and end of the circular beam the 4th natural frequencies reduction are almost same and when crack is

located near free end of the beams the 4th natural frequencies are generally unharmed although when the depth of crack is quite more.

Case (3):- Comparison between Uniform Beam of Rectangular and Circular cross-sections with Multiple Cracks

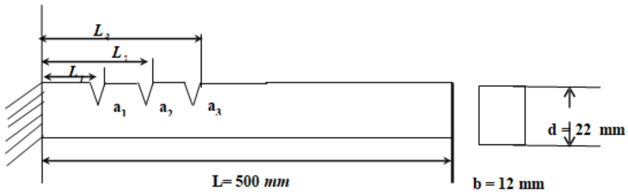


Figure 4.28: Sketch of multiple cracked uniform cantilever beam of rectangular cross-section

Location of Cracks for both rectangular (R) and circular beams (C):
Case 1: $L_1/L=0.1$, $L_2/L=0.2$, $L_3/L=0.3$
Case 2: $L_1/L=0.6$, $L_2/L=0.7$, $L_3/L=0.85$
Case 3: $L_1/L=0.25$, $L_2/L=0.50$, $L_3/L=0.75$
For all the cases**:** $a_1/d=0.2$
$a_2/d=0.3$

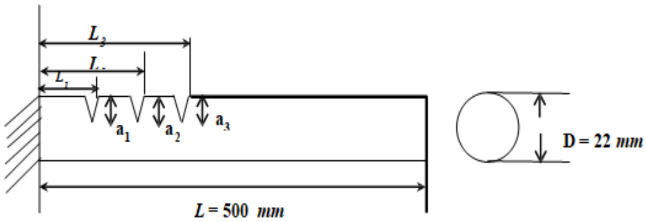


Figure 4.29: Sketch of multiple cracked uniform cantilever beam of circular cross-sectio

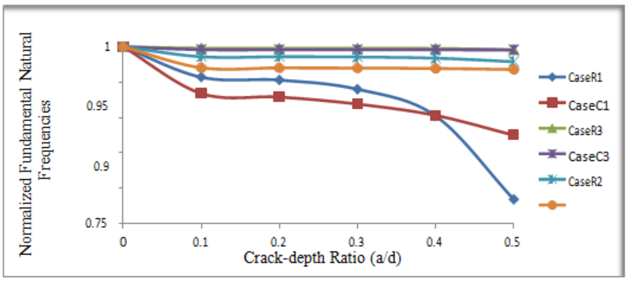


Figure 4.30: Comparison of normalized fundamental natural frequencies of multiple cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Fig4.30, it is observed that when cracks are located near the fixed end of the beam the normalized fundamental natural frequencies of multiple cracked beams of both rectangular and circular cross-sections are high to that of cracks located at center and at free end of the beam. When fractures are positioned at free end of the beam the fundamental natural frequencies are generally unharmed although when the depth of crack is quite more as in case of single cracked beams. It is also observed that in case of rectangular cross-section the fundamental normalized natural frequencies reduction is less than that of circular cross-section for all the cases respectively except when crack is located near the fixed end, crack depth ratio is 0.5 where the difference in frequency reduction is 10

beams, when crack- depth ratio is 0.5 the frequency reduction of rectangular beam is more by 8% than circular beam. When crack is located near free end fixed end of the beams they show similar pattern of variation In Figure4.32 it is observed that in case of rectangular cross-section the 3rd natural frequencies difference is high than that of circular cross-section for all the cases respectively. When crack is located at near fixed end or at center or near free end of the beam the rectangular beam will show more variation than circular beam for all the crack positions.

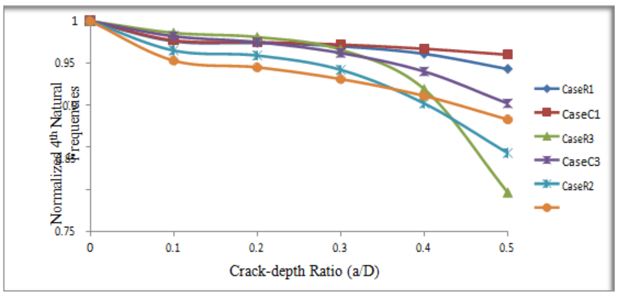


Figure 4.33: Comparison of normalized 4th natural frequencies of multiple cracked beams of rectangular and circular cross-sections with respect to crack-depth ratio.

From Figure4.33, it is observed that for crack locations at center and near free end of the beam the 4th natural frequency reduction is higher for rectangular section than circular section when the crack depth ratio is 0.5. However we find changes in the frequency reduction when crack-depth ratio is 0.4.

STEPPED BEAM WITH MULTIPLE CRACKS

Case (1):- Effect of Step present in Uniform, Single stepped and Two stepped beams of Rectangular cross-sections

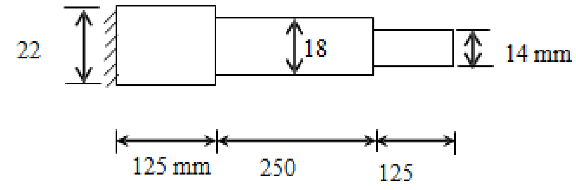


Figure 4.34: Sketch of Two stepped cantilever beam

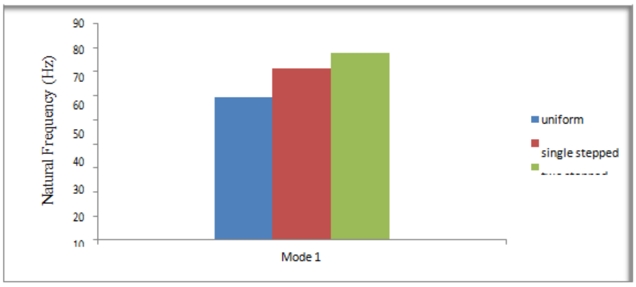


Figure 4.35: Plot of fundamental natural frequency (Hz) variation of uniform, single stepped and two stepped beams with respect to Mode1

From Figure4.35, it is observed that there is step wise increase in the fundamental frequency variation for uniform, single stepped and two stepped beams by 17.08% and 23.97% for single stepped and two stepped beams with respect to uniform beam respectively.

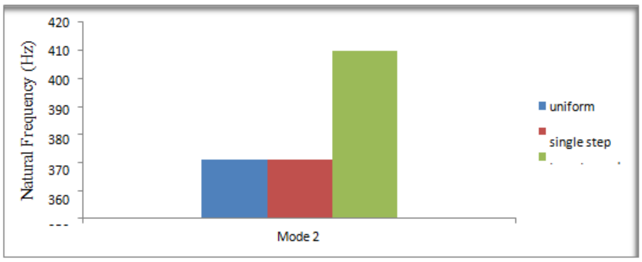


Figure 4.36: Plot of 2nd natural frequency (Hz) variation of uniform, single stepped and two stepped beams with respect to Mode 2 In Figure4.36 it is observed that the 2nd natural frequency for single stepped beam is increased by very less percentage 0.014% whereas for two stepped beam the increase is 9.41% when compared to uniform beam respectively.

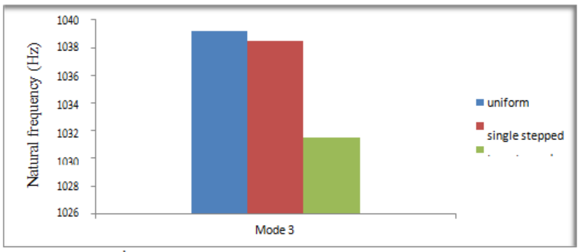


Figure 4.37: Plot of 3rd natural frequency variation (Hz) of uniform, single stepped and two stepped beams with respect to Mode 3

From Figure4.37, it is observed of 3rd natural frequency is high for uniform beam and the difference with respect to single stepped beam is reduced by 0.067%. For two stepped beam frequency reduction is 0.73% compared to uniform beam.

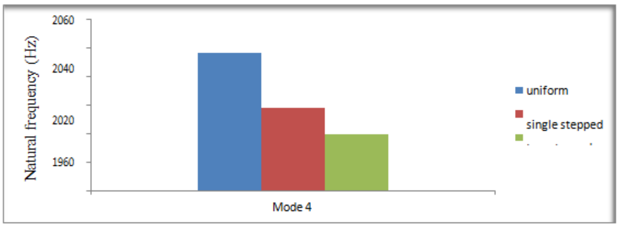


Figure 4.38: Plot of 4th natural frequency (Hz) variation of uniform, single stepped and two stepped beams.

In Figure4.38 it is observed that there is step wise decrease in the 4th natural frequency for uniform, single stepped and two stepped beams. There is decrease by 1.90% and 2.81% for single stepped and two stepped beams with respect to uniform beam respectively.

From Figure 4.35, 4.36 for Mode1 and Mode 2 the natural frequency increases as step is present in the beam whereas from Figure 4.37, 4.38 for Mode3 and Mode 4 the natural frequency decreases.

Case (2):- Effect of Step present in Single Cracked Beam of Rectangular cross-section

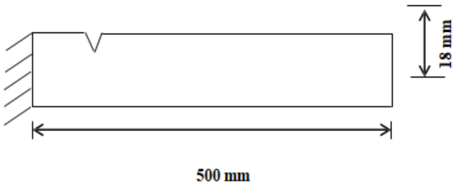


Figure 4.39: Sketch of Uniform cantilever beam with single crack of rectangular cross-section

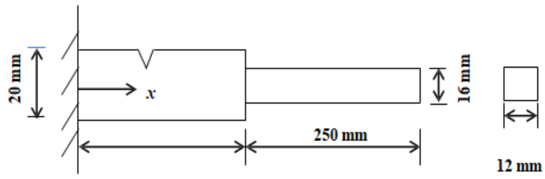


Figure 4.40: Sketch of single step cantilever beam of rectangular cross-section

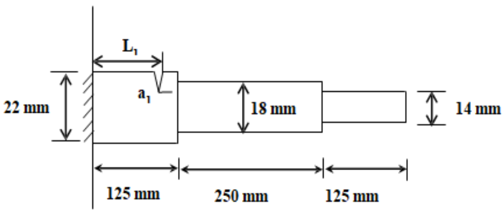


Figure 4.41: Sketch of two stepped cantilever beam of rectangular cross-section with single crack

Case (a): L_1/L=0.2; Crack is located near the free end of the beams

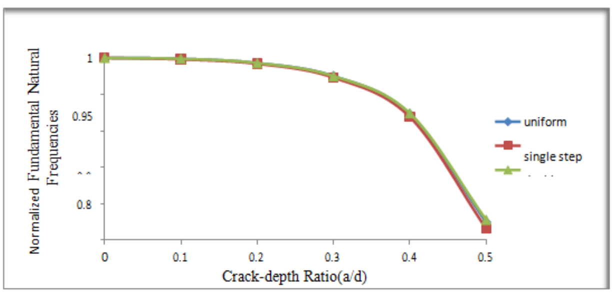


Figure 4.42(a): Comparison of normalized fundamental natural frequencies of uniform, single stepped and two stepped rectangular beams of single crack with respect to crack-depth ratio

From Figure4.42(a) it is observed that there is no much variation in normalized fundamental natural frequencies for uniform, single stepped and double stepped rectangular beams with single crack where all the beams exhibit the same pattern.

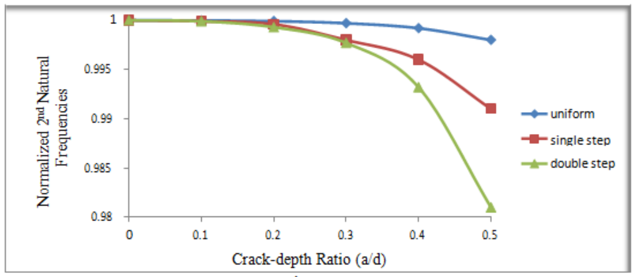


Figure 4.43(a): Comparison of normalized 2nd natural frequencies of uniform, single stepped and two stepped rectangular beams with single crack.

From Figure 4.43(a) it is observed that uniform, single stepped and double stepped rectangular beams with single crack follow ascending pattern however the frequency reduction of normalized 2nd natural frequencies is less for all the beams. The frequency reduction of two stepped beam is more by 1.7% to that of uniform beam.

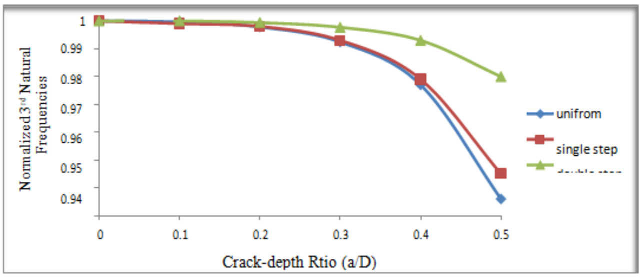


Figure 4.44(a): Comparison of normalized 3rd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack.

From Figure 4.44(a) it is observed that normalized 3rd natural frequencies of uniform, single stepped and double stepped rectangular beams with single crack follow descending pattern but the variation between uniform and stepped beams is high. The frequency reduction of two stepped beam is less by 5% to that of uniform beam

IV. CONCLUSIONS

The presence of number of cracks, crack location, crack-depth ratio the analysis of dynamic properties of the beam is done by finding the natural frequencies. The following conclusions are drawn from the present investigation of the uniform and stepped beams subjected to vibrate freely with multiple cracks using finite element analysis by using Finite Element Method (FEM) in MATLAB environment.

- A detailed formulation is presented for free vibration of uniform and stepped beam with multiple transverse open cracks.
- The frequency reduction increases as the crack-depth ratio increases for all the modes irrespective of uniform beam or stepped beam.
- Crack located closer of the fixed end of the beams in all cases frequency reduction variation

is significant than crack closer to the free end of the beam even when the crack-depth ratio is relatively high.

- Crack located closer of the free end of the beams in all cases will have higher effect on the 4th natural-frequencies than crack closer to the fixed end of the beam.
- Crack located at the center of the uniform beam will have higher 3rd natural frequency reduction.
- For all the beams either uniform beams or stepped beams irrespective of number of steps as the number of cracks present in the beam enhances, frequency occurred naturally of the beams reduces for extact crack-depth ratio due to reduction of stiffness.
- The frequency reduction is higher for uniform rectangular beam than uniform circular beam for both beams having same moment of inertia.
- When crack is located at near fixed end or at center or near free end of the beam i.e., for any crack location along the length of the beam the 3rd natural frequency reduction for rectangular beam is more than circular beam.
- Irrespective of number of cracks present in the beam the uniform beams show similar pattern of variation for all the normalized frequencies.
- Positions of cracks present along the length of the beam across uniform, single step, two stepped beams the major variation in the frequency reduction starts when crack-depth ratio is 0.3 and increases up to crack-depth ratio 0.5.
- For uniform beam or stepped beam with single or multiple cracks when located near the free end of the beam the fundamental frequency reduction is highest.

From the above discussions, it is clear that cracks cause the reduction of natural frequency. The presence of multiple cracks weakens the beam from the point of view of reduction in natural frequency. So cracks play a critical role on the vibration behaviour of the structures. The vibration behaviour of cracked circular and rectangular uniform as well as stepped beams is influenced by the geometry, material, location and size of cracks. The figures dealing with variation of the frequencies are recommended for identification of crack location and intensity for uniform and stepped beams. The above recommendations for design of beams are valid within the range of geometry and material considered in this study. So the designer has to be careful while dealing with structures subjected to cracks. This can be used to the advantage of design of stepped beams. The vibration characteristics of the cracked beams can be used as a tool for structural health monitoring, identification of crack location and extend of damage in beams and also helps in assessment of structural integrity of the structures.

SCOPE FOR FUTURE STUDY

- The complete analysis of the current work is carried out based on the Bernoulli-Euler beam structure and it can be extended for Timoshenko beam based structure for hygrothermal effects.
- Comparison of the analytical results of present analysis can be done with experimental results using FFT Analyser.
- The present study can be extended to study the effects of various parameters such as natural frequencies and bending modes for multi-cracked stepped beams of circular cross-sections.
- The study of free vibrational analysis of beams presently done can be extended by studying the buckling analysis of stepped beams.
- This study can be extended to study the variations in the dynamics parameters of the composite stepped beams with multiple cracks.

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