CONSTRUCTION OF AUTOMATIC EQUIPMENT SYSTEM FOR DRIVING VESSELS ON THE WATER SURFACE

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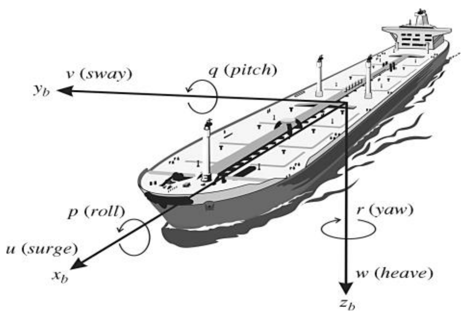
ABSTRACT: In order to carry out the construction of kinetic equations for unmanned ships on the water, the content of the article refers to the definition of the reference systems as variables of motion such as position, velocity, angular velocity of ships in coordinate systems. By analyzing the movements, the position of the ship, which is concerned with the geometric aspect of the motion, from which the kinetic equations system for the unmanned ship on the water surface will be presented as a tissue model math picture.

1. QUESTION

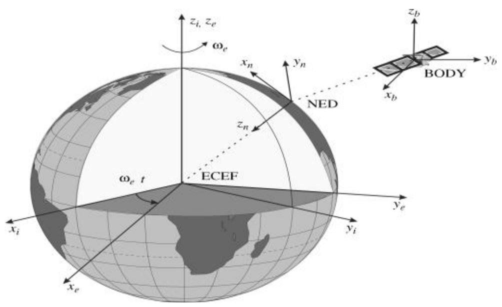
Vietnam is a coastal country with the sea three times the size of the mainland. A series of companies such as wharves, drilling rigs, oil pipes ... All of the above tasks require a device capable of automatically performing large workloads. One solution is to use an unmanned ship. This widget has automatic action. In this format, we are only interested in the geometric plane and the motion progression in geometric form.

2. COMBINATION SYSTEM AND MULTIPLAYER BETWEEN BRANCH SYSTEM

2.1. Reference systems



Picture 1. The velocities of the six degrees of freedom: u, v, w, p, q, r in the frame of reference attached to the ship $\{b\} = (x_b,\ y_b,\ z_b)$



Picture 2. The ECEF frame of reference $\{e\} = (x_e,\ y_e,\ z_e)$

rotates with an angle ωe compared with the ECI frame of reference $\{i\} = (x_i,\ y_i,\ z_i)$

For ships, the motion will have the first three coordinate variables and its time derivative, respectively, will represent position and movement changes in the x, y, and z axes; The last three coordinate variables and its time derivative represent the ship's direction and rotation, respectively (Picture 1).

In addition, it is possible to define the above six coordinate variables with another way called surge, sway, heave, roll, pitch, yaw as shown in picture 2.

The frame of reference in which the earth is centered has two types: ECI and ECEF.

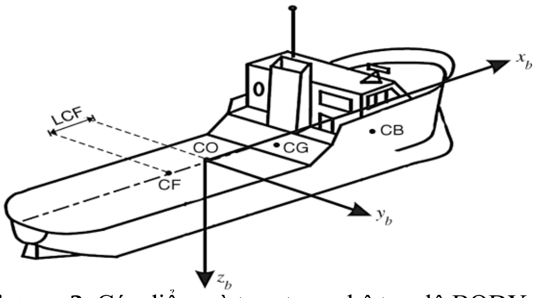
ECI (The Earth-Centered Inertial) is the inertial reference system for navigation on the ground. The symbol is {i}. The root of {i} lies at the center Oi of the earth, along with the axes shown in Figure 2.

ECEF (The Earth-Centered Earth-Fixed) is a revision of ECI. This is the frame of reference that considers the earth to be fixed (not rotating). The symbol is {e}. The origin of {e} is still at the center of the earth, but its axis is rotated away from the ECI (rotation angle ω_e = 7.2921 x 10^{-5}*rad/s*). The {e} coordinate system is commonly used in global navigation, navigation and control.

Geographical reference system also has two types: NED and BODY.

NED (North-East-Down) denotes {n} a coordinate system with center On is defined as a plane tangent to the surface of the earth. The position of {n} relative to {e} will be determined through the two angles l and μ representing the longitude and latitude respectively.

BODY is a moving coordinate system fixed to the ship. The symbol is {b} with root Ob. The position and direction of the vessel are described respectively in the {e} or {n} reference systems, while the angular velocity and linear velocity of the vessel should be expressed in the BODY coordinate system. The base Ob is usually chosen to coincide with the midpoint of the boat in the floating line above the water, that point is CO in figure 3.



Picture 3. Các điểm và trục trong hệ tọa độ BODY

Symbols in the 6th order system:

Above, have fully defined the frames of reference, next we will define the vector symbols corresponding to the frames of reference:

A vector without coordinate system when considered in the {n} coordinate system is represented:

$$\vec{u} = u_1^n \vec{n}_1 + u_2^n \vec{n}_2 + u_3^n \vec{n}_3$$

with: $\vec{n}_i$ (i = 1, 2, 3) – vector units in the {n} coordinate system

u_1^n – projection size of $\vec{u}$ on the way $\vec{n}_i$

We will also use the standard un coordinate form of vector in {n} through the column vector form as follows:

$$u^n = \left[u_1^n, u_2^n, u_3^n\right]^T \in R^3$$

When analyzing ship motion, it is unavoidable to avoid cases of coordinated conversion between {b}, {n}, {e} coordinate systems. The following symbols will then be defined to solve this problem:

$v_{\frac{b}{n}}^e$ – the linear velocity of the point O_b compared with {n} represented in {e}

$\omega_{\frac{n}{e}}^{e}$ – the origin velocity of {n} relative to {e} shown in {b}

f_b^n – force with line acting through point Ob represented in {n}

m_b^n – moment of the point Ob represented in {n}

Θ_{nb} – Corner Euler $\{n\}$ and $\{b\}$

Thus, the variable definitions are shown in Table 1.

Table 1. Định nghĩa các biến trong hệ sáu bậc tự do

Tier free	Motion description	Force and moment	Linear velocity and angular velocity	Location and corner Euler
1	Movement in the direction *x* (surge)	*F*	*u*	*x*
2	Movement in the direction *y* (sway)	*Y*	*v*	*y*
3	Movement in the direction *z* (heave)	*Z*	*w*	*z*
4	Rotates around the axis *x* (roll, heel)	*K*	*p*	ϕ
5	Rotates around the axis *y* (pitch, trim)	*M*	*q*	θ
6	Rotates around the axis *z* (yaw)	*N*	*r*	ψ

Also need to define the following elements:

Location ECEF: $p_{\frac{b}{e}}^{e} = \begin{bmatrix} x \\ y \\ x \end{bmatrix} \in R^3$; Longitude and latitude: $\Theta_{en} = \begin{bmatrix} l \\ \mu \end{bmatrix} \in S^2$

Location NED: $p_{\frac{b}{n}}^{n} = \begin{bmatrix} N \\ E \\ D \end{bmatrix} \in R^3$; Trạng thái (góc Euler): $\Theta_{en} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \in S^3$

Linear velocity BODY: $v_{\frac{b}{n}}^{b} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} \in R^3$; Original velocity BODY: $\omega_{\frac{b}{n}}^{n} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \in R^3$

Force BODY: $f_b^b = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \in R^3$; Momen BODY: $m_b^b = \begin{bmatrix} K \\ M \\ N \end{bmatrix} \in R^3$

In which, R^3 is a third Euclidean space and S^2 is a quadratic torus (meaning that there are two angles defined in the interval [0; 2π]) and S3 is a third sphere.

At this point, the general motion of the six-degree of freedom ship with O_b can be viewed as the origin depicted by the following vectors:

$$\eta = \begin{bmatrix} p_{\underline{b}}^{n} \ hay \ p_{\underline{b}}^{e} \\ n \quad\quad n \end{bmatrix} \in R^3 \text{x } S^3; \ \upsilon = \begin{bmatrix} v_{\underline{b}}^{b} \\ n \\ \omega_{\underline{b}}^{b} \\ n \end{bmatrix} \in R^6; \ \tau = \begin{bmatrix} f_b^b \\ m_b^b \end{bmatrix} \in R^6$$

Where, η denotes the location and direction vector of the train, υ denotes the linear velocity and origin velocity, and denotes the force and torque acting on the train.

2.2. Rotation matrix, transition between frames of reference

Normally, if considering Location, it is often used NED coordinate system, but when considering force, torque or speed, the BODY coordinate system should be used. Therefore, analyzing the transition between frames of reference is very necessary. One of the tools to solve this conversion problem is rotation matrix.

The rotation matrix R between the two frames of reference a and b is denoted R_b^a and is a matrix belonging to a set of special orthogonal groups of order 3 SO (3):

$$SO(3) = \{R \mid R \in R^{3x3}, \ R \text{ là trực giao và có } \det R = 1\} \quad (1)$$

Here, SO (3) is a subset of all 3rd order orthogonal matrices, meaning SO (3) $\in$ O (3) with:

$$O(3) := \{R \mid R \in R^{3x3}, RR^T = R^T R = I\} \quad (2)$$

The rotation matrix $R \in SO(3)$ and satisfy the nature:

$$RR^T = R^T R = I, \det R = 1 \quad (I \textit{ is the unit matrix})$$

The inverse matrix of the rotation matrix is: $R^{-1} = R^T$. From here, the symbol for converting a vector from one frame of reference to another is as follows:

$$v^{to} = R_{from}^{to} v^{from} \quad (3)$$

Inside, $v^{from} \in R^3$ vector representation to be converted to new coordinate system is performed by rotation matrix R_{from}^{to} and the result is vector $v^{to} \in R^3$.

3. CONSTRUCTION OF A KINEMATIC EQUATION FOR A SHIP

Derived from Euler's theorem "Any change in relative orientation between two undistorted bodies or two frames of reference {A} and {B} can be provided by a simple rotation of {B} in {A} ".

Consider an internal velocity vector BODY $v_{\underline{b}}^{b}$ (n) and an internal velocity vector NED $v_{\underline{b}}^{n}$ (n). According to the above theorem, can be expressed $v_{\underline{b}}^{n}$ (n) according to the $v_{\underline{b}}^{b}$ (n) through a simple rotation, call $\lambda = [\lambda_1 \ \ \lambda_2 \ \ \lambda_3]^T$ is the unit vector ($\|\lambda\| = 1$) of that axis and β is the angle at which the NED needs to rotate. At that time, we write:

$$v_{\underline{b}}^{b} = R_b^n v_{\underline{b}}^{b}, \ R_b^n := R_{\lambda\beta} \quad (4)$$
$$n \qquad\quad n$$

In which, the matrix $R_{\lambda\beta}$ can be calculated as a rotation matrix representing an angle around the axis as follows:

$$R_{\lambda\beta} = I_{3x3} + \sin\beta.S(\lambda) + [1 - \cos\beta]S^2(\lambda) \quad (5)$$

Where I is the unit matrix, $S(\lambda)$ is the symmetry matrix and $S^2(\lambda) = S(\lambda)$, $S(\lambda) = \lambda^{\lambda^T} - I_{3x3}$ because λ is the unit vector.

Expanding (7) we have:

$$R_{\lambda\beta} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & ` \end{bmatrix} + \sin\beta \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix} + (1-\cos\beta) \begin{bmatrix} \lambda_1^2 - 1 & \lambda_1\lambda_2 & \lambda_1\lambda_3 \\ \lambda_1\lambda_2 & \lambda_2^2 - 1 & \lambda_2\lambda_3 \\ \lambda_1\lambda_3 & \lambda_2\lambda_3 & \lambda_3^2 - 1 \end{bmatrix}$$

With the elements of $R_{\lambda\beta}$:

$$\begin{aligned}
R_{11} &= (1-\cos\beta)\lambda_1^2 + \cos\beta \\
R_{22} &= (1-\cos\beta)\lambda_2^2 + \cos\beta \\
R_{33} &= (1-\cos\beta)\lambda_3^2 + \cos\beta \\
R_{12} &= (1-\cos\beta)\lambda_1\lambda_2 - \sin\beta\lambda_3 \\
R_{21} &= (1-\cos\beta)\lambda_1\lambda_2 + \sin\beta\lambda_3 \\
R_{13} &= (1-\cos\beta)\lambda_1\lambda_3 + \sin\beta\lambda_2 \\
R_{31} &= (1-\cos\beta)\lambda_1\lambda_3 - \sin\beta\lambda_2 \\
R_{23} &= (1-\cos\beta)\lambda_2\lambda_3 - \sin\beta\lambda_1 \\
R_{32} &= (1-\cos\beta)\lambda_2\lambda_3 + \sin\beta\lambda_1
\end{aligned}$$

3.1. Euler's angle variation

The corners Euler: roll(ϕ), pitch(θ) and yaw(ψ) used for BODY velocity vector analysis $v_{\frac{b}{n}}^{b}$ in the frame of reference NED.

$R_b^n(\Theta_{nb}): S^3 \rightarrow SO(3)$ denotes the Euler angular rotation matrix with the argument $\Theta_{nb} = [\phi, \theta, \psi]^T$. So:

$$v_{\frac{b}{n}}^{n} = R_b^n(\Theta_{nb}) v_{\frac{b}{n}}^{b} \quad (7)$$

Considering the main rotation matrices (revolving around one axis of the coordinate system), obtained the following results:

According to the axis x, $\lambda = [1 \;\; 0 \;\; 0]^T$ and $\beta = \phi$. Apply formula (6) with replacement $\lambda = [1 \;\; 0 \;\; 0]^T$ and $I = \phi$ ando:

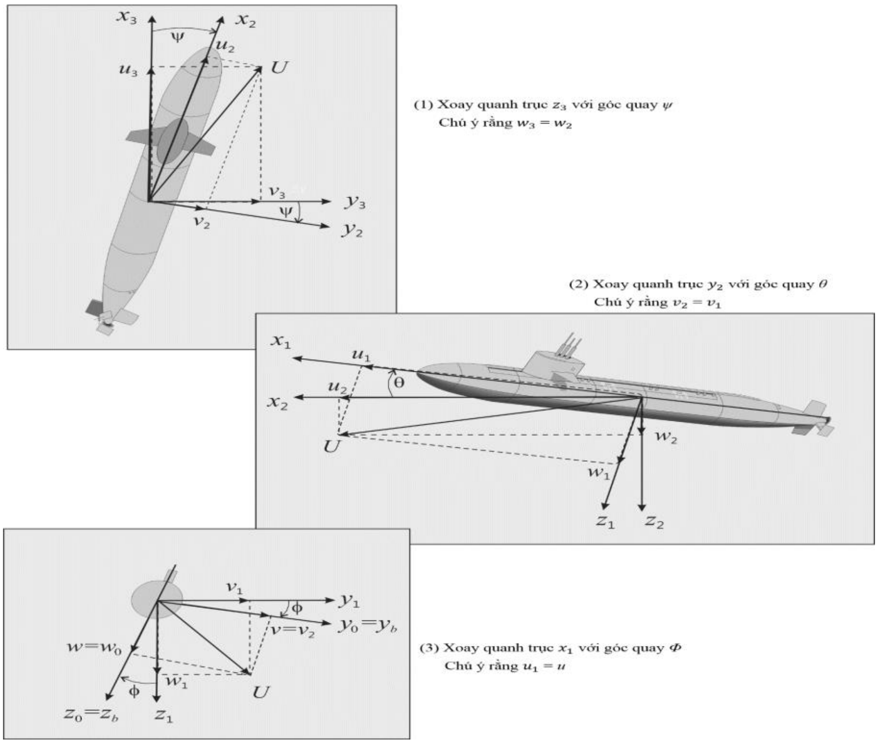
$$R_{x,\phi} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \quad (8)$$

– According to the axis y, $\lambda = [0 \;\; 1 \;\; 0]^T$ and $\beta = \theta$. axis z then $\lambda = [0 \;\; 0 \;\; 1]^T$ and $\beta = \psi$:

$$R_{y,\phi} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad (9)$$

$$R_{z,\phi} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (10)$$

These transitions can be clearly seen in Figure 4



Picture 4. Euler's angle rotation order (zyx convention) of a submarine rotated from {n} to {b} using key rotations

Continue, denoting $R_b^n(\Theta_{nb})$ as key rotation matrices. We have NED and BODY as two independent separate frames of reference, starting here to perform a rotation for the NED coordinate system with $R_{z,\psi}$, Then we continue to do one rotation $R_{y,\theta}$ and finally the rotation $R_{x,\phi}$. The results we obtained:

$$R_b^n(\Theta_{nb}) = R_{z,\psi} R_{y,\theta} R_{x,\phi} \quad (11)$$

The order of performing the above rotations according to the zyx convention. In navigation, positioning and control, the zyx convention from {n} to {b} is often used. In addition, thanks to property 1 of rotation matrix, it is easy to define rotation matrix from {b} to {n} as follows: $R_n^b(\Theta_{nb}) = R_b^n(\Theta_{nb})^{-1} = R_b^n(\Theta_{nb})^T = R_{x,\phi}^T R_{y,\theta}^T R_{z,\psi}^T$ (12)

To better understand this problem, study Figure 4 with the following assumptions:

For $x_3y_3z_3$ is the coordinate system obtained after shifting the coordinate systems NED $x_ny_nz_n$ parallel to itself until its original coincides with the BODY coordinate system. Coordinates $x_3y_3z_3$ will rotate a yaw angle (ψ) around the axis z_3 to create a new coordinate system $x_2y_2z_2$. Then the coordinate system $x_2y_2z_2$ continue to rotate a pitch angle (θ) around y_2 to create a new coordinate x1y1z1. Finally, the x1y1z1 coordinate system rotates at an angle roll (ϕ) x_1 to finally coincide with the coordinate system BODY $x_by_bz_b$.

Expansion (11) obtains the matrix $R_b^n(\Theta_{nb})$ The elements are calculated according to the desired coordinate variables ϕ, θ, ψ, in the end we have:

$$R_b^n(\Theta_{nb}) = \begin{bmatrix} \cos\psi\cos\theta & -\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi & \sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta \\ \sin\psi\cos\theta & \cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi & -\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi \\ -\sin\theta & \cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \quad (13)$$

The velocity vector can now be represented $v_{b/n}^{b}$ in {n} is as follows:

$$\dot{p}_{b/n}^{n} = v_{b/n}^{n} = R_b^n(\Theta_{nb}) v_{b/n}^{b} \quad (14)$$

Expansion (14) yields:

$$\dot{N} = u\cos\psi\cos\theta + v(-\sin\psi\cos\phi + \cos\psi\sin\theta\sin\phi) + w(\sin\psi\sin\phi + \cos\psi\cos\phi\sin\theta)$$
$$\dot{E} = u\sin\psi\cos\theta + v(\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi) + w(-\cos\psi\sin\phi + \sin\psi\sin\theta\cos\phi)$$
$$\dot{D} = -u\sin\theta + v\cos\theta\sin\phi + w\cos\theta\sin\phi \quad (15)$$

From (14) it is easy to infer:

$$v_{b/n}^{b} = R_b^n(\Theta_{nb})^{-1}\,\dot{p}_{b/n}^{n} = R_b^n(\Theta_{nb})^{T}\,\dot{p}_{b/n}^{n} \quad (16)$$

3.2. Angular velocity variation

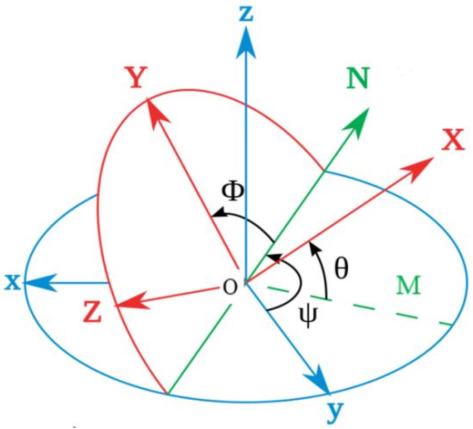
Call the angular velocity vector conversion matrix BODY $\omega_{b/n}^{b} = [p,\, q,\, r]^T$ and derivative state vector $\Theta_{nb} = [\dot{\phi},\, \dot{\theta},\, \dot{\psi}]^T$ là $T_\Theta(\Theta_{nb})$ with the following transformation formula:

$$\Theta_{nb} = T_\Theta(\Theta_{nb})\omega_{b/n}^{b} \quad (17)$$

Now, will find a way to calculate $T_\Theta(\Theta_{nb})$. $\omega_{b/n}^{b}$ it is not possible to integrate directly to obtain the actual coordinate angles. However, it is possible to turn to vector analysis $\Theta_{nb} = [\phi,\, \theta,\, \psi]^T$ with the attention that these angles do not represent a general coordinate system (since they are Euler angles), we will calculate $T_\Theta(\Theta_{nb})$ through $T_\Theta^{-1}(\Theta_{nb})$ instead of calculating directly by considering in many cases the following:

$$\omega_{b/n}^{b} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R_{x,\phi}^T \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R_{x,\phi}^T R_{y,\phi}^T \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = T_\Theta^{-1}(\Theta_{nb})\dot{\Theta}_{nb} \quad (18)$$

Split case and compute $T_\Theta^{-1}(\Theta_{nb})$ will be visualized more easily through figure 5



Picture 5. Angles representations Euler (α, β, γ) between the two coordinate systems xyz and XYZ have the same origin O

In Figure 5, ON is the intersection of the two planes xOy and ZOY and OM is the intersection of xOy and XOY. Then, considering the correlation, XYZ is BODY and xyz is NED. Rotate the xyz around z at an angle yaw ψ (then the new coordinate system has the y axis coinciding with ON. Then, rotate around this ON axis an angle θ. At this point, the x-axis of the new coordinate system will coincide with X. Finally, rotating around X by an angle we will have the final coordinate system coinciding with XYZ. Clearly we see ϕ, θ, ψ do not belong to the same general coordinate system. Through the above rotation we see that angle θ will be considered after the rotation $R_{x,\phi}$. Therefore, it is possible to switch corners θ about the same coordinate system as ϕ by $R_{x,\phi}^{-1} = R_{x,\phi}^{T}$. The argument is similar to angle ψ We will obtain the above formula.

Expansion (18), get the end result for $T_{\Theta}\left(\Theta_{nb}\right)$:

$$T_{\Theta}\left(\Theta_{nb}\right) = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{bmatrix} \tag{19}$$

Obviously with the calculation as above we see, will not exist $T_{\Theta}\left(\Theta_{nb}\right)$ with angle pitch $\theta = \pm 90^{0}$ and at the same time $T_{\Theta}\left(\Theta_{nb}\right)$ also does not satisfy the properties of the rotation matrix ie $T_{\Theta}^{-1}\left(\Theta_{nb}\right) \neq T_{\Theta}^{T}\left(\Theta_{nb}\right)$.

$T_{\Theta}\left(\Theta_{nb}\right)$, (17) deployment obtained:

$$\begin{aligned} \dot{\phi} &= p + q\sin\phi\tan\theta + r\cos\phi\tan\theta \\ \dot{\theta} &= q\cos\phi - r\sin\phi \\ \dot{\psi} &= q\frac{\sin\phi}{\cos\theta} + r\frac{\cos\phi}{\cos\theta}, \quad \theta \neq \pm 90 \end{aligned} \tag{20}$$

Combining (15) and (20) to form the hexagonal kinetic equation for ships, we have:

$$\dot{p}_{b/n}^{n} = \begin{bmatrix} R_{n}^{b}\left(\Theta_{nb}\right) & O_{3x3} \\ O_{3x3} & T_{\Theta}\left(\Theta_{nb}\right) \end{bmatrix} \begin{bmatrix} \upsilon_{b/n}^{b} \\ \omega_{b/n}^{b} \end{bmatrix} \tag{21}$$

4. CONCLUDE

Thus, a system of six-fold kinetic equations for unmanned ships on water has been built. Next, will build a dynamic equation for the train to know how the impact of forces and torque on the train will affect the speed and acceleration of the train. These results will help in the design and simulation of controls and navigation for unmanned ships on water.

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