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ASSIMILATION OF VARIOUS LDPC CODING SCHEME

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Abstract: Low-density parity-check (LDPC) codes can be measured as serious competitors to turbo codes in requisites of performance and intricacy. For making the efficient use of available limited bandwidth, code modulation is the good competitor on comparing it with some other. In coding theory nowadays Low density parity-check codes are one of the most modern topics and has very fast encoding and decoding algorithms .LDPC are right striking mutually theoretically and practically. The throughout outlook of LDPC coding technique is present in this paper. Here we will familiar with some noticeable encoding terminologies including Regular LDPC codes, Irregular LDPC codes, Bi-layer LDPC codes and Punctured LDPC codes. We will see that irregular LDPC perform better than regular LDPC thus are more desirable and among all punctured LDPC codes require less design complexity.

Index Terms: Gallager codes, low density parity check codes, Belief propagation, regular/irregular codes, bilayer and punctured codes.

I. INTRODUCTION

LOW density parity-check code (LDPC) is an error correcting code used in noisy communication channel to lessen the likelihood of damage of information. With this property of LDPC codes data transmission rate can be as close to Shannon's limit. Low-density parity-check codes, introduced by Gallager in 1962 [1] and their performance under "belief propagation" decoding, has been the theme of recent experimentation and study [2, 3, 4, 5]. The interest in these codes stems from their near Shannon limit performance. The connection between transmitter and receiver is established through communication channel. The communication channel takes place through wire as well as wireless medium or can use optical channels. The other medium such as optical disc, magnetic tapes and discs etc. can also be called as communication channel because they can also carry data through them. LDPC was reinvented. Low-Density Parity-Check (LDPC) codes are known to be powerful due to their capacity-approaching property for single user communication channels. Received data steadfastness depends on the medium of channel used and external noise too. Noise effect is very dominant in medium and possibly will introduced somber error in transmitted data. According to Shannon's theorem if data rate is less than that of channel capacity, data could be transmitted unfailingly. In this theorem a sequence of codes less than the channel capacity have a capability as the code length goes to infinity [6]. LDPC have made its way into some modern applications such as Wi-MAX, 10GBase-T Ethernet, Wi-Fi, and Digital Video Broadcasting (DVB). Due to encroachment of VLSI technologies, it is conceivable to manufacture very high speed embedded circuits. Such circuits are used in today's communication sector. Even ULSI technologies are use in various applications nowadays. High speed computer and potent software design tools are accessible.

II. SYSTEM MODEL

In this system model, one relay R is measured, which is on the direct line between a source S and a destination D, as shown in Fig. 1. The relay system drives in a half-duplex mode. Consequently, the signal xS is transmitted from the source to both the relay and the destination in the first time block of length t and the signal xR is transmitted from the relay to the destination while the source keeps silent in the second time block of length (1−t), where t is defined as the time-division factor. The distance between the source and the destination is normalized to 1 and $0 \le d \le 1$ denotes the distance between the source and the relay. By defining the noise terms nSR, nRD and nSD for the SR, RD and SD links, respectively, the receive signal ySR, yRD and ySD are given by:

$$ySR = xS + nSR$$
$$ySD = xS + nSD$$
$$yRD = xR + nRD$$

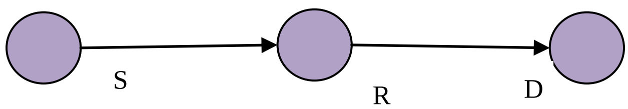


Figure 1: Relay system between Source and destination

The overall relay channel is defined as the source-destination link including the relay, which contains all the components of the network that form the end-to-end connection. In comparison to a classical link, i.e., a system without a relay, the theoretical limit of the decode-and-forward strategy for the overall relay channel is enhanced. Fig. 2 illustrate the different SNR capacities for the classical link and for the overall relay channel for BPSK transmission and SR distance d=0.5. Capacity curve for the overall relay channel is shown with respect to the SNR on the SD link, i.e., SNRSD. The capacity of the overall relay link be subject on the capacities of the SD, SR, and RD links and is given by [7]:

$$C = \sup_{0 \le t \le 1} \min\{t \cdot CSR, t \cdot CSD + (1 - t) \cdot CRD\}$$

It can be observed that the capacity of the overall relay link is 0.4 bits/s/Hz for SNRSD = −4 dB as indicated by the cross. In order to achieve the same throughput on a classical link, the required SNR is SNR* = −1.3 dB as indicated by the circle. This SNR* is defined as the effective SNR and represents the SNR a classical link would essential to attain the identical capacity as the relay channel. Apparently, there is a one-to-one relationship between SNRSD and SNR* since both curves increase monotonically in Fig. 2.

III. CONSTRUCTION OF LDPC CODES

A. Regular LDPC Codes

The edifice scheme proposed by Gallager resides of establishing a sparse parity check matrix H by arbitrarily defining the positions of '1's, with a static number of one's '1's per column and per row, thus crafting a regular LDPC code. The situation on the number of '1's per column and per row can be relaxed, provided that the number of '1's per column s, satisfies s > 2. Here, the LDPC code is thought to be irregular. The conditions to be satisfied in the construction of the parity check matrix H of a binary regular LDPC code are [8]

- The corresponding parity check matrix H should have a fixed number v of '1's per row.
- The corresponding parity check matrix H should have a fixed number s of '1's per column.
- The overlapping of '1's per column and per row should be at most equal to one. This is a necessary condition for avoiding the presence of cycles in the corresponding bipartite graph.
- The parameters s and v should be small numbers compared with the code length.

It is yet very challenging to gratify the third condition if the objective is to construct good LDPC codes, since cycles are unavoidable in the bipartite graph of an efficient LDPC code [9]. The above construction does not ordinarily lead to the design of a sparse parity check matrix H of systematic form, and so it is usually obligatory to exploit Gaussian elimination to alter this resulting matrix into a systematic parity check matrix H' = [In−k PT], where In−k is the identity sub matrix of dimension (n − k) × (n − k). The initially designed sparse parity check matrix H is the parity check matrix of the LDPC code, whose generator matrix G is of the form G = [P Ik].

Summarizing the design method for an LDPC code, a sparse parity check matrix H = [AB] is constructed first, obeying the corresponding construction conditions. In general, this initial matrix is not in systematic form. Sub-matrices A and B are sparse. Sub-matrix A is a square matrix of dimension (n − k) × (n − k) that is non-singular, and so it has an inverse matrix A−1. Sub-matrix B is of dimension (n − k) × k. The Gaussian elimination method, operating over the binary field, modifies the matrix H = [A B] into the form H' = [Ik A−1 B] = [Ik PT]. This operation is equivalent to pre-multiplying H = [A B] by A−1. Once the equivalent parity check matrix H' has been formed. The corresponding generator matrix G can be constructed by using the sub-matrices obtained, to give G = [P Ik]. In this way both the generator and the parity check matrices are defined, and the LDPC code is finally designed. Note that the matrices of interest are H and G.

LDPC codes can be categorized, according to the construction method used for generating the corresponding sparse parity check matrix H, into [10]

- random LDPC codes and
- structured LDPC codes

Generally, random LDPC codes appearance a marginally better BER performance than that of structured LDPC codes, but these latter codes are much less complex to encode than the former codes. The construction methodology suggested by MacKay [4, 11] is random, while other approaches comprise those based on finite field geometries, balanced incomplete block designs and cyclic or quasi-cyclic structures [10, 12].

B. Irregular LDPC Codes

An irregular LDPC code is one with a sparse parity check matrix H that has a flexible number of '1's per row or per column. In general, the BER performances of irregular LDPC codes are superior to those of regular LDPC codes. There are several construction methods for irregular LDPC codes [8].

C. Bilayer LDPC codes for the relay channel

A single source X tries to communicate to a single destination Y with the help of a relay in a relay channel. The relay receives Y1 and sends out X1 based on Y1.The relay channel is defined by the joint distribution p(y, y1|x, x1). $\tilde{X}$(w) have the graphical code structure as shown in Fig. 3, with k1 zero parity check bits and k2 extra parity check bits generated by the relay. The source data rate is (n-k1)/n and the source's code words are enforced to satisfy k1 zero parity check bits. The relay decodes the source's codeword based on the first k1 parity bits and generates k2 extra parity bits which are then transmitted to the destination by means of a distinct codebook, i.e., X1(s). The destination first decodes the k2 extra parity bits sent by the relay and then decodes the source's codeword knowing that it should satisfy k1 zero parity bits and k2 extra parity bits generated by the relay. In order to incorporate this protocol, we consider a bilayer structure for the (n,k1+k2) LDPC code for $\tilde{X}$(w). In this bilayer structure, one layer corresponds to a (n,k1) capacity approaching LDPC code (for the source-relay channel) consisting of the k1 zero parity bits and a second layer consists of the k2 extra parity bits which modifies the first layer in a way that the overall (n,k1 + k2) LDPC code is capacity attaining for the source-destination channel.

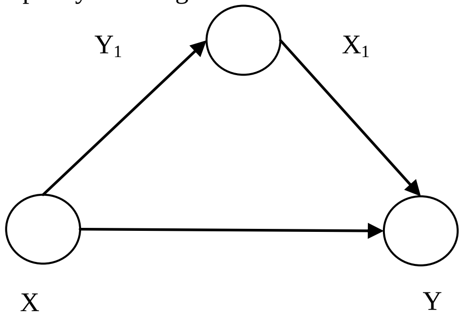


Figure 3: The relay channel

D. Bilayer LDPC Codes

Let the LDPC code for $\widetilde{X}(w)$ have n variable nodes, k1 check bits and an additional k2 check bits (generated by the relay for the destination.) As shown in Fig. 4, The solid part correspond to the subgraph and represents a LDPC code designed for the channel between the source and the relay. The relay decodes the subgraph code and provide extra parity check bits for the destination. The destination decodes the transmitted codeword over the overall hypergraph. The graph corresponding to this code consists of two layers. The left-graph or sub graph which is directly connected to the left k1 parities and represents the code designed for the source-relay channel. The right-graph is defined to be the part of the graph that is directly connected to the right parties. The right-graph is designed so that it modifies the subgraph in such a way that the resulting hypergraph guarantees successful decoding at the destination. By discriminating left edges to be those edges that are connected to the left k1 parities from right edges that are connected to the right k2 parities, it can be seen that from each variable node, two types of edges may emanate. Therefore, for each variable node of the graph two different degrees are conceivable: the left degree which is defined to be the number of left edges connected to the variable and the right degree which is defined to be the number of right edges that are connected to variables.

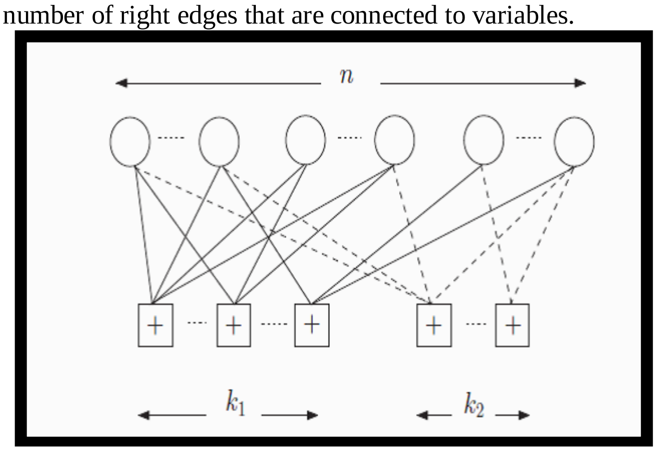


Fig.4: Bilayer LPDC codes.

Let λi,j be the variable degree distribution of hypergraph defined as the percentage of edges in the hypergraph which are connected to a variable node of degree (i,j), i.e., the percentage of edges that have left degree i and right degree j. Note that i≥ 2 since no variable of degree less than 2 is allowed in the subgraph and j ≥0 as some of the nodes may only be connected to the left parities. For given λi,j's satisfying $\sum_{2\leq i,0\leq j} \lambda_{i,j}$= 1 and for a specific set of check degrees, both the subgraph and the hypergraph can be constructed. In standard LDPC code design, it is common to fix one or at most two different values for check degrees. Some guidelines for choosing appropriate check degrees can be found in [13] and [14]. Fixing check degrees, a bilayer code design problem can be formulated as that of finding a doubly indexed distribution λi,j such that the induced subgraph is capacity approaching at SNR1 and the overall hypergraph is capacity approaching at SNR2 < SNR1.

The degree distribution of the subgraph code can be found as a linear combination of λi,j as follows:

$$Vi = \frac{1}{\eta}\sum_{j\geq 0} \frac{i}{i=j} \lambda_{i,j}$$

Where 0 < ŋ < 1 is the ratio of the total number of edges in the subgraph and the total number of edges in the hypergraph. Assuming a fixed number of check nodes with fixed degrees, the total number of edges in the subgraph and the hypergraph are fixed and therefore ´ is a constant.

The coefficients vi's are related to the code rate between the source and the relay. Let E be the total number of edges in the subgraph. Then, the block length of the code, which is equivalent to the total number of variable nodes in the graph, is given by E $\sum_{i\geq 2} v_i/i$; and there are E $\sum_{i\geq 2} vp_i/i$ left parity check nodes (where p_i's denote the fixed left check degree distribution.) Hence, the rate of the source-relay code is given by:

$$R = 1 - \frac{\sum_{i\geq 2} p_i/i}{\sum_{i\geq 2} v_i/i}$$

A capacity approaching code for the decode and forward strategy should have an appropriate degree distribution $\lambda_{i,j}$ that maximizes the above rate. The two-dimensional degree distribution for variable nodes on an edge perspective is defined as

$$\lambda^E(\omega, z) = \sum_{i=2}^{d_{v,1}} \sum_{k=0}^{d_{v,2}f} \lambda^E_{i,k}\, \omega^{i-1} z^{k-1}$$

Where dv,1 and dv,2 represent the maximum values of variable degrees in H1 and H2, respectively. Note that k starts at 0 since variable nodes that have no connections with H2 are allowed.

To jointly design H1 and H, H1 is still treated as a single user LDPC code, whose variable nodes receives the mean LSR from the SR link, as shown in Fig. 5(left). For the overall code decoded at the destination, the bi-layer density evolution is briefly introduced; more details can be found in [15]. For a variable node vi in H,ML,i is defined as the set of left check nodes cL connected to vi and MR,i is defined as the set of right check nodes cR connected to vi. The updating rule at the variable node vi is

$$L(v_i^q) = \sum_{\substack{\ell\in M_{L,i} \\ \ell\neq q}} L(c_{L,\ell}) + \sum_{\ell\in M_{R,i}} L(c_{R,\ell}) + LSD \,.\, ySD$$

If the outgoing message will flow to a left check node cL,q. Similarly, if the outgoing message will be fed to a right check node cR,q, the updating rule at the variable node vi is

$$L(v_i^q) = \sum_{\ell\in M_{L,i}} L(c_{L,\ell}) + \sum_{\substack{\ell\in M_{R,i} \\ \ell\neq q}} L(c_{R,\ell}) + LSD \,.\, ySD$$

where LSD = $\frac{2}{\sigma^2_{SD}}$ = 2SNRSD is the LLR from the direct SD link. For a left check node cL,j, KL,j is
defined as the set of variable nodes in H connected to cL,j. The updating rule at the left check node cL,j is

$$c_{L,j}^{\sim q} = \tanh\frac{L(c_L,j)}{2} = \prod_{\substack{l\in K_{L,j} \\ l\neq q}} \tanh\frac{L(v_1)}{2}$$

if the destination of the outgoing message is a variable node vq. The set of variable nodes in H connected to cR,j is denoted as KR,j for a right check node cR,j. Subsequently, if the outgoing message will be passed to a variable node vq, the updating rule at the right check node cR,j is

$$c^{\sim q}_{R,j} = \tanh\frac{L(c_{R,j})}{2} = \prod_{\substack{l\in K_{R,j}\\ l\neq q}} \tanh\frac{L(v_1)}{2}$$

Where LRD = $\frac{2}{\sigma^2_{SD}}$ = 2SNRSD is the channel reliability of the RD link. Note that in contrast to the left check nodes, the right check nodes also receive the LLR LRD _ yRD from the RD link, as can be observed in above and Fig. 5(right).

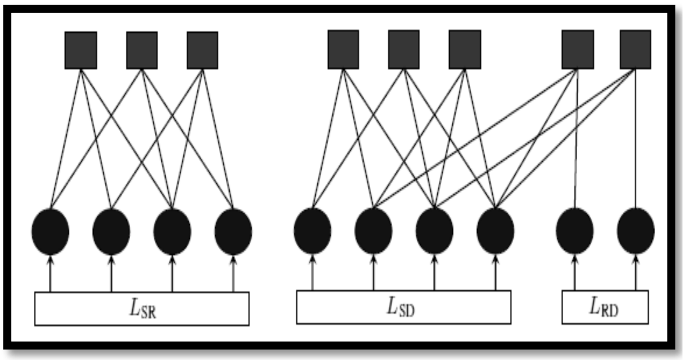
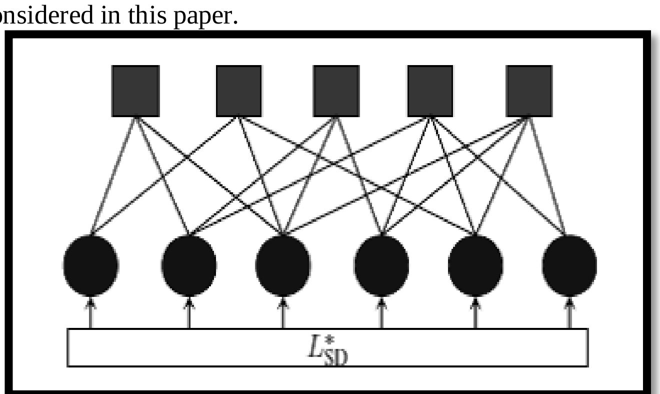


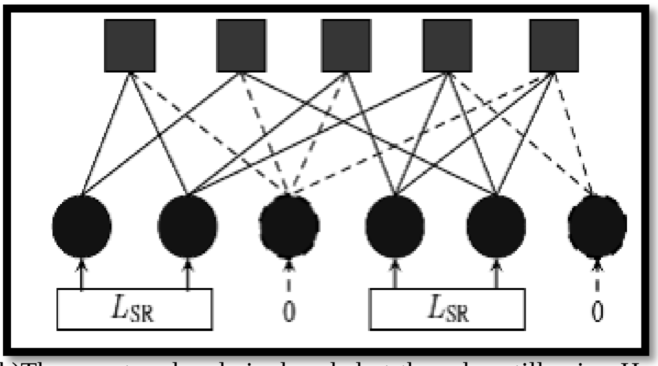
Fig.5. Factor graph of the source code and overall code for the code optimization of Bi-layer LDPC codes

IV. PUNCTURED LDPC CODES

In this section, we study some fundamental properties of punctured LDPC codes. Puncturing is one of the most common methods to construct rate-compatible codes. In this method, to change the rate of a code to a higher rate, we puncture (delete) a subset of the code word bits. The design process of both the previous schemes calls for joint optimization of two LDPC codes based on density evolution. To avoid the complicated derivation and calculation, we propose a much simpler scheme to design distributed. LDPC codes using puncturing, as done, e.g., in for convolutional codes. First, a mother code H with code rate Rc is designed either by linear programming or by using the codes from Urbanke's website [17] as a single-user LDPC code to suit the channel condition of the overall relay channel. Then a proportion of parity bits are punctured out randomly in order to meet the channel condition of the SR link. The code rate is raised to Rc,S due to puncturing. Note that there exist other sophisticated puncturing patterns that achieve better performances [18], [19] but only random puncturing is considered in this paper.



(a)The mother code is decoded at the destination using H, where all the variable nodes receive L^*_{SD} as the overall relay channel is treated equivalently as a direct link.



(b)The punctured code is decoded at the relay still using H, where the un-punctured variable nodes receive LSR and the punctured nodes (- -) receive 0 LLR value.

Fig.6. Punctured LDPC codes

Figure 6. Illustrates the process above. The whole graph is designed by taking L^*_{SD} from the overall relay channel. Then some bits are punctured out, which are represented by the dashed parts in Fig. 6(b). The un-punctured variable nodes receive LSR from the SR link while the punctured nodes receive zero LLR values and get recovered as the decoding iteration at the relay goes on. The bits punctured at the source are now added by the relay, and the destination receives the whole codeword, but with different SNRs, i.e., SNRSR for the source code and SNRRD for the additional parity bits. Note that the simplicity of this scheme locates on the fact that the design only depends on the channel condition of the overall relay channel, and just the puncturing probability is adapted to the certain relay position. This is an indicator that there is no complex joint optimization of two codes. Furthermore, the same decoder H is used at both the relay and the destination for punctured LDPC codes.

V. CONCLUSION

It is quite clear that punctured LDPC codes require much less design complexity compared with the other schemes. Both of LDPC codes based on a single-user scenario and Bi-layer LDPC codes call for joint optimization of two codes that are strongly connected (one being the subgraph of the other). Different decoders are also needed at the relay and the destination. For punctured LDPC codes, only one standard single-user LDPC code has to be optimized as a mother code. Subsequently, some bits are punctured out to form the source code. The same decoder can also be used at both the relay and the destination. Furthermore, both LDPC codes based on a single-user scenario and Bilayer LDPC codes have to be designed for each triple of SNRs, namely, SNRSR, SNRSD and SNRRD, related to a relay network, whereas the punctured LDPC codes just have to be optimized for the overall relay channel. Therefore, much fewer codes have to be designed for the same amount of scenarios. This leads to much lower complexity. Comparisons of the these schemes with respect to their performance show, that punctured LDPC codes are superior in comparison with Bi-layer LDPC codes when the relay is near the destination and vice versa. LDPC codes based on a single-user scenario show the best performance for the assumed complexity restrictions. When a performance complexity tradeoff is required, LDPC codes based on a single-user scenario as well as punctured LDPC codes seem to be promising candidates, whereas Bi-layer LDPC codes seem to require exhaustive design complexity. In order to analyze the performance of bilayer LDPC codes, the bilayer density evolution is developed as an extension of the conventional density evolution. For specific channel parameters, it is demonstrated that a bilayer LDPC code can achieve the theoretical decode-and forward rate of the relay channel to within a 0.19 dB gap to the source-relay channel capacity and a 0.34 dB gap to the relay-destination channel capacity[20].

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