

## OPTIMAL PLACEMENT OF PHASOR MEASUREMENT UNIT (PMU) USING LINEAR PROGRAMMING

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**ABSTRACT:** This report describes a study undertaken to determine the optimal locations of Phasor measurement units (PMUs) for a given power system. Power systems are rapidly becoming populated by PMUs. PMUs have multiple uses at substations. They provide valuable Phasor information for protection and control of power systems during abnormal operation. Under normal operations, they also help in monitoring the system state. This project focuses on the use of PMU measurements in state estimators. The principle objective was to investigate methods of determining optimal locations for PMUs so that the system state of an entire power system can be observable. Several factors affect how this can be accomplished, such as the available data from existing conventional measurements, the number and location of zero injection buses, the number and location of installed PMUs, and, of course, the system topology. A numerical procedure where the problem is formulated as an integer optimization problem. The solution algorithm was implemented in a computer program. Several case studies were conducted to evaluate the algorithm's performance. Fewer PMUs may be needed if there are zero injection buses in the system. The software is developed in Matlab using an external optimization program (TOM LAB) which is used to implement the integer programming solution. The results of the project can be extended to investigate the benefits of adding a small number of PMUs at strategic locations to improve bad data detection and identification capability in the system.

### I. INTRODUCTION

Secure operation of power systems requires close monitoring of the system operating conditions. This is traditionally accomplished by the state estimator which resides in the control centre computer and has access to the measurements received from numerous Substations in the monitored system. By collecting analog measurements and the status data of the circuit breakers from remotely monitored and controlled substations and feeding them as input into state estimation function, state estimation can provide an estimate for all metered and un-metered electrical quantities and network parameters of the power system, detect and filter out gross errors in the measurement set and detect the topology errors in the network configuration. Until recently, available measurement sets did not contain phase angle measurements due to the technical difficulties associated with the synchronization of measurements at remote locations. Global positioning satellite (GPS) technology alleviated these difficulties and lead to the development of phasor

measurement units(PMU).Synchronized Phase Measurement Unit (PMU) is a monitoring device,which was first introduced in mid-1980s. Phasor measurement units (PMU) are devices, which use synchronization signals from the global positioning system (GPS) satellites and provide the phasors of voltage and currents measured at a given substation. As the PMUs become more and more affordable, their utilization will increase not only for substation applications but also at the control centers for the EMS applications. One of the applications, which will be significantly affected by the introduction of PMUs, is the state estimator.State estimators provide optimal estimates of bus voltage phasors based on the available measurements and knowledge about the network topology. These measurements are commonly provided by the remote terminal units (RTU) at the substations and include real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents. More recently, synchronized phasor measurements have started to become available at selected substations in the system. One of the issues faced by the planning engineers is how to select the best locations to install new PMUs.

The pure sinusoidal waveform  $x(t) = X_m \cos(\omega t + \phi)$  is commonly represented as a phasor  $X = X_r + jX_l = \left(\frac{X_m}{\sqrt{2}}\right) + e^{j\phi}$  where  $\phi$  depends on the definition of the time scale. For this Standard, this basic concept is adapted as the representation of power system Sinusoidal signals.

The synchrophasor representation X of a signal x(t) is the complex value given by

$$X = X_r + jX_l = \left(\frac{X_m}{\sqrt{2}}\right) + e^{j\phi}(\cos\phi + j\sin\phi)$$

Where  $(X_m/ \sqrt{2})$  is the rms value of the signal x(t) and  $\phi$  is its instantaneous phase angle relative to a cosine function at nominal system frequency synchronized to UTC. This angle is defined to be 0 degrees when the maximum of x(t) occurs at the UTC second rollover (1 PPS time signal), and -90 degrees when the positive zero crossing occurs at the UTC second rollover. Figure illustrates this relationship.

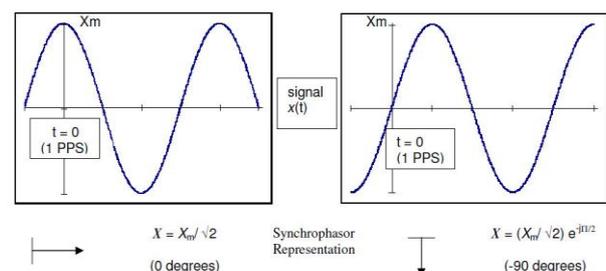


Figure 1 Convention for synchrophasor representation

II. PHASOR MEASUREMENT UNIT

A PMU is a device which measures the electrical waves on an electricity grid, using a common time source for synchronization. Time synchronization allows synchronized real-time measurements of multiple remote measurement points on the grid. In power engineering, these are also commonly referred to as synchphasors and are considered one of the most important measuring devices in the future of power systems. A PMU can be a dedicated device, or the PMU function can be incorporated into a protective relay or other device



Figure 2 Model 1690 Phasor Measurement Unit

PMU INSTALLATION AND CONNECTION:

Installation of a typical 10 Phasor PMU is a simple process. A phasor will be either a 3 phase voltage or a 3 phase current. Each Phasor will, therefore, require 3 separate electrical connections (one for each phase). We are talking about 6 wires per phasor – 2 for each phase of either voltage or current. The PMU will also measure the line frequency from a specific voltage phasor (typically a major bus assigned by the user). Typically an electrical engineer designs the installation and interconnection of a PMU at a substation or at a generation plant. Substation personnel will bolt equipment rack to the floor of the substation following established seismic mounting requirements. Then the PMU along with a modem and other support equipment will be mounted on the equipment rack. They will also install the GPS antenna on the roof of the substation per manufacturer instructions. The antenna signal cable will be connected to the antenna and brought directly to the PMU. Substation personnel will also install “shunts” in all Current Transformer (CT) secondary circuits that are to be measured. Potential Transformer (PT) connections will not require the installation of any additional equipment other than terminal blocks and fuses. They will have to run wires from the CT shunts and the PTs to either an interface cabinet or directly to the input connections of the PMU. Each phasor (either Voltage or Current) will require three connections – one for each phase.

In addition to the CT and PT connections the PMU will also require the following

Connections:

- Power connection – typically from station batteries.
- Station ground connection.
- Global Positioning Satellite (GPS) antenna connection.
- Communication circuit connection (Modem if using 4-wire connection or Ethernet for network connection).

After all the connections are made, the PMU is configured and tested. This task is typically performed by a substation Test Technician. The utility’s IT department will play a key role will the phasor data connections phase of the PMU installation. After the entire input channel configuration and testing is completed, the PMU is connected to the utility’s Phasor Data Concentrator (PDC) via 4-wire Modem or Ethernet connection depending on the bandwidth needs. They will also need to evaluate the need to install additional communication equipment in order to provide the necessary circuit connections between the PDC at the master site and the PC workstations at the client sites.

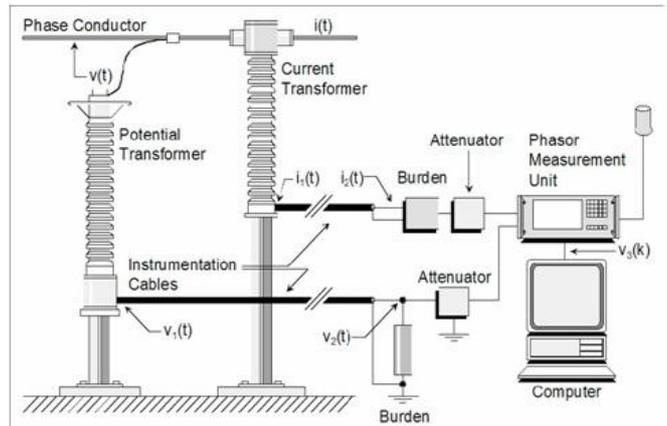


Figure 3 Phasor Measurement installation and connection

Working of PMU

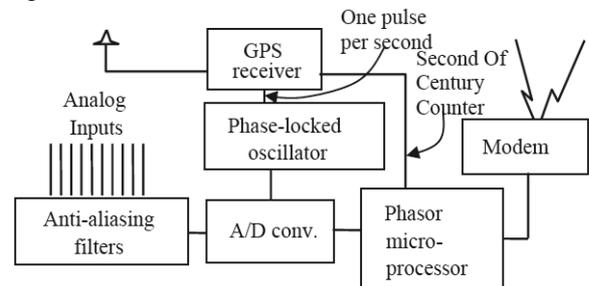


Figure 4 Block Diagram of Phasor Measurement Unit

The PMUs manufactured by different manufacturers differ from each other in many important aspects. It is therefore difficult to discuss the PMU hardware configuration in a way which is universally applicable. However, it is possible to discuss a generic PMU, which will capture the essence of its principal components. Figure 4 is based upon the configuration of the first PMUs built at Virginia Tech (and shown in Figure 4). Remember that PMUs evolved out of the development of the “symmetrical component distance relay”. Consequently the structure shown in Figure 4. Parallels that of a computer relay. The analog inputs are currents and voltages obtained from the secondary windings of the current and voltage transformers. All three phase currents and voltages are used so that positive-sequence measurement can be carried out. In contrast to a relay, a PMU may have currents in several feeders originating in the substation and voltages belonging to various buses in the substation. The current and voltage signals are converted to voltages with

appropriate shunts or instrument transformers (typically within the range of  $\pm 10$  volts) so that they are matched with the requirements of the analog-to-digital converters. The sampling rate chosen for the sampling process dictates the frequency response of the anti-aliasing filters. In most cases these are analog-type filters with a cut-off frequency less than half the sampling frequency in order to satisfy the Nyquist criterion. As in many relay designs one may use a high sampling rate (called oversampling) with corresponding high cut-off frequency of the analog anti-aliasing filters. This step is then followed by a digital 'decimation filter' which converts the sampled data to a lower sampling rate, thus providing a 'digital anti-aliasing filter' concatenated with the analog anti-aliasing filters. The advantage of such a scheme is that the effective anti-aliasing filters made up of an analog front end and a digital decimation filter are far more stable as far as aging and temperature variations are concerned. This ensures that all the analog signals have the same phase shift and attenuation, thus assuring that the phase angle differences and relative magnitudes of the different signals are unchanged. The sampling clock is phase-locked with the GPS clock pulse (to be described in the following section). Sampling rates have been going up steadily over the years – starting with a rate of 12 samples per cycle of the nominal power frequency in the first PMUs to as high as 96 or 128 samples per cycle in more modern devices, as faster analog-to-digital converters and microprocessors have become common place. Even higher sampling rates are certainly likely in the future leading to more accurate phasor estimates, since higher sampling rates do lead to improved estimation accuracy [1]. The microprocessor calculates positive sequence estimates of all the current and voltage signals. Earlier, certain other estimates of interest are frequency and rate of change of frequency measured locally, and these also are included in the output of the PMU. The time-stamp is created from two of the signals derived from the GPS receiver. This will be considered in greater detail in the next section. For the moment, it is sufficient to say that the time-stamp identifies the identity of the "universal time coordinated (UTC) second and the instant defining the boundary of one of the power frequency periods as defined in the IEEE standard to be considered below. Finally, the principal output of the PMU is the time-stamped measurement to be transferred over the communication links through suitable modems to a higher level in the measurement system hierarchy.

### III. HIERARCHY FOR PHASOR MEASUREMENT SYSTEMS

The PMUs are installed in power system substations. The selection of substations where these installations take place depends upon the use to be made of the measurements they provide. The optimal placement of PMUs will be considered in some of the following chapters which discuss some of the applications of phasor measurements. In most applications, the phasor data is used at locations remote from the PMUs. Thus an architecture involving PMUs, communication links, and data concentrators must exist in order to realize the full

benefit of the PMU measurement system. A generally accepted architecture of such a system is shown in Figure 5

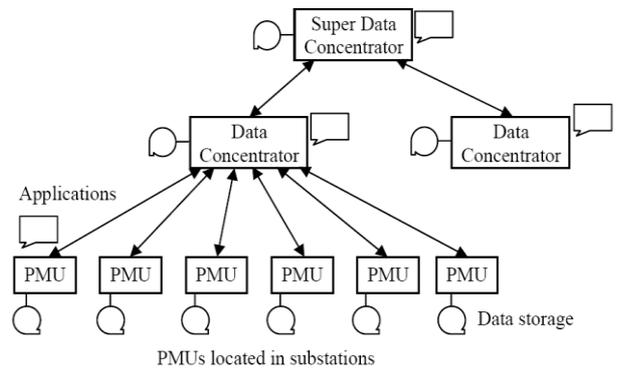


Figure 5 Hierarchy for Phasor measurement systems

In Figure 5 the PMUs are situated in power system substations, and provide measurements of time-stamped positive-sequence voltages and currents of all monitored buses and feeders (as well as frequency and rate of change of frequency). The measurements are stored in local data storage devices, which can be accessed from remote locations for post-mortem or diagnostic purposes. The local storage capacity is necessarily limited, and the stored data belonging to an interesting power system event must be flagged for permanent storage so that it is not overwritten when the local storage capacity is exhausted. The phasor data is also available for real-time applications in a steady stream as soon as the measurements are made. There may well be some local application tasks which require PMU data, in which case it can be made available locally for such tasks. However, the main use of the real-time data is at a higher level where data from several PMUs is available. The devices at next level of the hierarchy are commonly known as "phasor data concentrators" (PDCs). Typical function of the PDCs is to gather data from several PMUs, reject bad data, align the time-stamps, and create a coherent record of simultaneously recorded data from a wider part of the power system. There are local storage facilities in the PDCs, as well as application functions which need the PMU data available at the PDC. This can be made available by the PDCs to the local applications in real time. (Clearly, the communication and data management delays at the PDCs will create greater latency in the real-time data, but all practical experience shows that this is not unmanageable. One may view the first hierarchical level of PDCs as being regional in their data-gathering capability. On a system wide scale, one must contemplate another level of the hierarchy (Super Data Concentrator in Figure 5). The functions at this level are similar to those at the PDC levels – that is, there is facility for data storage of data aligned with time-tags (at a somewhat increased data latency), as well as a steady stream of near real-time data for applications which require data over the entire system. Figure 5 shows the communication links to be bidirectional. Indeed, most of the data flow is upward in the hierarchy, although there are some tasks which require communication capability in the reverse direction. Usually these are commands for configuring the downstream components, requesting data in a particular form, etc.

IV. PLACEMENT ALGORITHM

The PMU devices are expensive to install and maintain on the electrical grid. Their cost can range from \$40,000 - \$60,000 including installation. The problem is to minimize PMU placement to minimize cost..The goal of placement algorithm is to achieve full system observability with a minimum number of PMUs thereby reducing cost. A suitable methodology is needed to determine the optimal location of in power system. Here a numerical method based on integer programming will be presented. The formulation of the problem facilitates analysis of network observability and is general enough to account for the existence of zero and non-zero power injections and power flow measurements.

Developed Method:

PMUs provide two types of measurements: bus voltage phasors and branch current phasors. Depending on the type of PMUs used the number of channels used for measuring voltage and current phasors will vary. In this report, it is assumed that each PMU has enough channels to record the bus voltage phasor at its associated bus and current phasors along all branches that are incident to this bus. The objective of the PMU placement problem is to render an observable system by using a minimum number of PMUs. An example of an optimally placed set of PMUs in a 14-bus system is shown Below in Figure 6 .

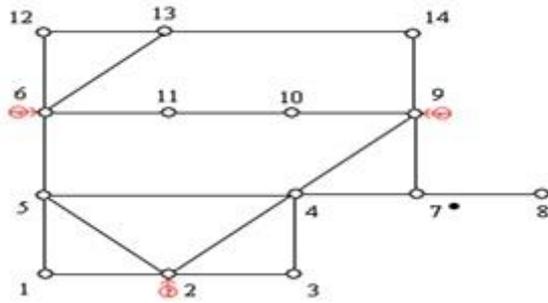


Figure 6 PMUs placed at buses 2, 6 and 9

Fig 6 In this system, there are three PMUs placed at buses 2, 6 and 9 respectively. Bus 7 is the only zero injection bus. The PMU at bus 2 can not only measure the voltage Phasor of bus 2, but also the current Phasors of branches 2-1, 2-3, 2-4 and 2-5. Using Ohm’s law, the voltage Phasors at buses 1, 3, 4 and 5 can be obtained from the branch currents and the voltage at bus 2. Having determined voltage Phasors at buses 1, 2, 3, 4, and 5, the current Phasors of branches 1-5, 3-4 and 4-5 can be calculated. Following the same logic, PMU at bus 6 can measure the voltage Phasor at bus 6 and the current Phasors of branches 6-5, 6-11, 6-12 and 6-13, thus allowing the calculation of the voltage phasors at buses 5, 11, 12, 13 and the current Phasor of branch 12-13. PMU at bus 9 can measure the voltage phasor at bus 9 and the current phasors of branches 9-4, 9-7, 9-10, 9-14 and allow the calculation of the voltage phasors at buses 4, 7, 10, 14, and the current phasors of branches 4-7. As voltage phasors of buses 10, 11, 13, 14 are known, current phaors of branches 10-11 and 13-14 can now also be calculated. Using the known current phasors of branches 4-7 and 9-7, and the zero injection at bus 7, the current phasor of branch 7-8 can be derived using the

Kirchhoff’s Current Law. The only remaining unknown voltage phasor at bus 8 can now be calculated by using the voltage phasor at bus 7 and the current phasor of branch 7-8. Thus the entire system becomes observable by placing only three PMUs at buses 2, 6, 9 and by considering the zero injection at bus 7. In this report, an integer based programming method is introduced to solve the PMU placement problem. This method will be discussed in detail via the use of the IEEE 14-bus example in the following sections. In Fig6 a numerical method based on Integer Programming will be presented to solve the optimal PMU placement problem. The formulation of problem is shown as below. For an n-bus system, the PMU placement problem can be formulated as follows:

$$\min \sum_i^n w_i . x_i \quad s.t \ f(X) \ge 1$$

Where X is a binary decision variable vector, whose entries are defined as

$$x_i = \begin{cases} 1 & \text{if a PMU is installed at bus } i \\ 0 & \text{otherwise} \end{cases}$$

Wi is the cost of the PMU installed at bus i.

f(X) is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise. 1^ is a vector whose entries are all ones. Inner product of the binary decision variable vector and the cost vector represents the total installation costs of the selected PMUs. Constraint functions ensure full network observability while minimizing the total installation cost of the PMUs. The procedure for building the constraint equations will be described for three possible cases where there are (1) no conventional measurement or zero injections, (2) flow measurements or (3) flow measurements as well as injection measurements (they may be zero injections or measured injections). Description of the procedure for each case will be given using IEEE 14-bus system example for clarification. However, the entire procedure is actually programmed and successfully tested on different size systems with diverse measurement configurations. Consider the IEEE 14-bus system and its measurement configuration shown in Figure 7 The black dot near bus 7 represents that bus 7 is a zero injection bus (or has a injection measurement installed) while the black box on line 5-6 represents a paired flow measurement on line 5-6.

V. A SYSTEM WITH NO CONVENTIONAL MEASUREMENTS AND/OR ZERO INJECTIONS

In this case, the flow measurement and the zero injection are ignored. In order to form the constraint set, the binary connectivity matrix A, whose entries are defined below, will be Formed first:

$$A_{k,m} = \begin{cases} 1 & \text{if } k = m \text{ or } k \text{ and } m \text{ are connected} \\ 0 & \text{otherwise} \end{cases}$$

Matrix A can be directly obtained from the bus admittance



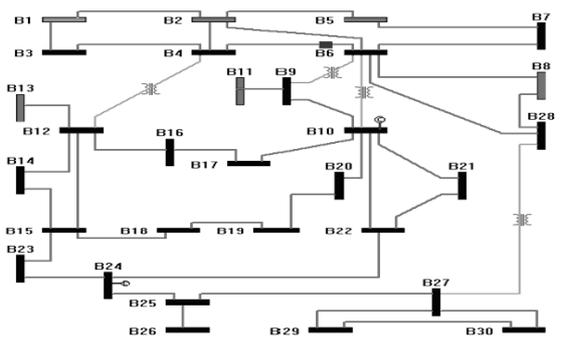


Figure 9 IEEE 30-bus systems

# of PMUs	Loc. (bus #)
32	2, 5, 9, 11, 12, 17, 21, 24, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 63, 68, 73, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114

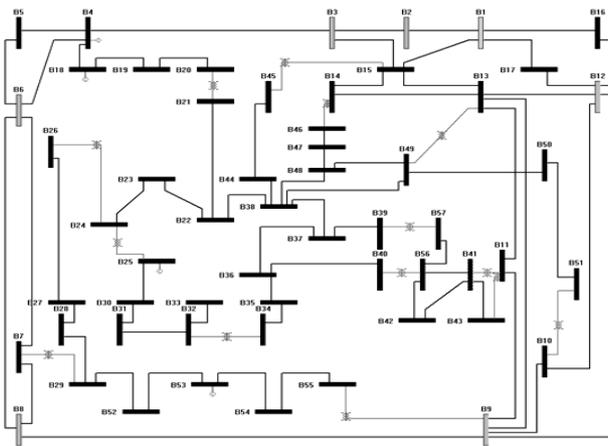


Figure 10 IEEE 57-buses System

Comparison with other method

system	Integer programming	Herustic approach	Simulated annealing
IEEE 14-bus system	4	4	4
IEEE 30-bus system	10	9	11
IEEE 57-bus system	17	19	18
IEEE 118-bus system	72	75	77

VII. CONCLUSION

From above thesis we found out that it is necessary to optimize the placement of PMU. PMU placement problem does not have a unique solution. Depending upon the starting point, the developed optimization scheme may yield different sets of optimal solutions, each one providing the same minimum number of PMUs but at different locations. On the other hand, it is not unusual to have additional considerations apart from strict observability criterion, when deciding on the location of PMUs.

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IEEE 118-bus system

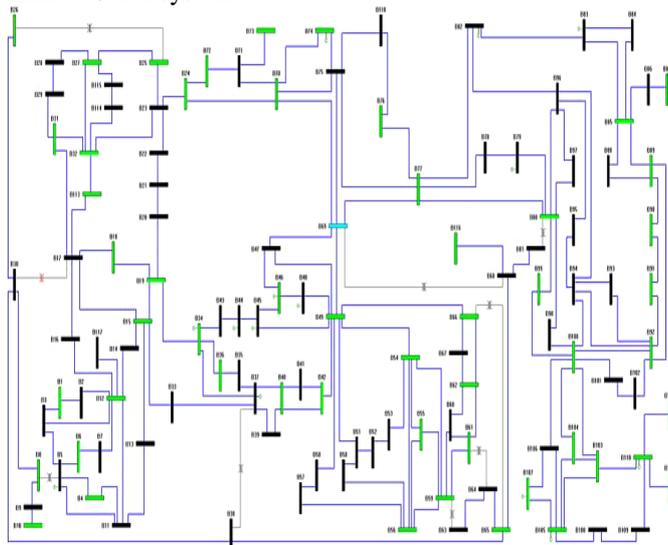


Fig 9 IEEE 118 Bus Test System

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