

QISKIT IMPLEMENTATION OF VARIOUS QUANTUM ERROR CORRECTION CODES IN QUANTUM INFORMATION PROCESSING SYSTEM**Dinesh Kumar Yadav¹, Shivender Goswami², Manoj Kumar^{3*}, Akash Rathor⁴, Ankit Chaudhary⁵**¹Department of Computer Science and Engineering, St. Andrews Institute of Technology and Management, Gurugram, Farrukhnagar, Haryana, India^{2,3,4,5}Department of Mathematics and Statistics, Gurukula Kangri (Deemed to be University), Haridwar, Uttarakhand, India.Email: ¹deantpo@saitm.ac.in, ²20522006@gkv.ac.in, ⁴20522002@gkv.ac.in, ⁵21522002@gkv.ac.in*Corresponding Author: ³manoj.kumar@gkv.ac.in

Abstract

Quantum error correction is a critical component in the construction of reliable quantum computing systems. In this study, we investigate various error correcting codes implemented on the Qiskit framework, a popular open-source quantum computing platform. We examine the performance of different codes, including the bit flip code for 3-qubit, phase flip code for 3-qubit, the five-qubit code, the Steane code, and the Shor code, under the influence of channel flip errors. Our objective is to analyze the logical error rate (LER) as a function of the total channel flip probability, providing insights into the efficacy of various error correcting codes in minimizing errors is a key aspect of interest in this study. To conduct our study, we simulate quantum error correction circuits on Qiskit, introducing a range of channel flip probabilities to mimic the existence of noise in practical quantum systems. Standard error correction techniques have been employed such as syndrome measurement and error detection to identify and correct errors. Analyzing the collected data, we construct a graph illustrating the relationship between the LER and the total channel flip probability for each code.

Keywords: *Quantum computing, quantum error correction codes, Qiskit, logical error rate, channel flip probability.*

Mathematics Subject Classification: 68Q12, 81P68

1.Introduction

The area of quantum information (q-information) theory has experienced remarkable advancements in technology, leading to the rapid and extraordinary growth of scalable quantum computation and communication. A fundamental aspect of this progress involves enhancing the theoretical aspects of robust and consistent q-information processing. In order to safeguard q-information from the detrimental effects of decoherence on quantum states, error control mechanisms are essential in q-communication. To address this, quantum error correction codes (QECC) are employed to safeguard from specific errors within the systems. Notably, these codes offer advantages even when encoding and recovery operations are imperfect. The development of quantum computing has yielded significant contributions, such as Shor's algorithm, which enables the factorization of large numbers in significantly very less computation time while comparing to classical computing [1-2]. Consequently,

quantum computers possess the capability to tackle computationally challenging problems with greater efficiency than as compared to classical computers [1-4]. Peter Shor's influential publication exploring error correction in q-system provides valuable insights into practical quantum computation. In his work, he introduces a technique for safeguarding delicate q-information from decoherence through encoding the information into a larger subspace of Hilbert space that surpasses its original dimensions [1]. Additionally, Calder Bank and Shor established asymptotic rates for code existence and derived upper bounds for these rates [4]. Error correction in q-system is a critical concept in the field of q-computing. Unlike classical computers, q-systems are highly sensitive to errors and noise caused by various factors such as environmental interactions and imperfections in hardware. Quantum error correction (QEC) techniques aim to safeguard q-information from these errors, allowing for more reliable and accurate computations. By implementing QECC, we can identify and correct errors that occur during q-operations. These codes encode the information in a larger space, introducing redundancy that helps in error detection and recovery. QEC enables us to preserve the delicate quantum states and mitigate the impact of noise, leading to improved accuracy and stability in q-computations.

Quantum computers offer great potential for solving complex computational problems that surpass the capabilities of classical computers. Nevertheless, the power of quantum computation comes with a drawback: quantum systems are highly vulnerable to errors resulting from noise and decoherence. Errors of this nature have the potential to disturb the fragile quantum information contained within qubits and undermine the dependability of quantum computations. QEC plays a crucial role in addressing this challenge. QEC is an essential technique that aims to safeguard quantum information by redundantly encoding it across a larger number of qubits [5]. Through the utilization of sophisticated error detection and correction codes, QEC enables the identification and rectification of errors, thereby preserving the integrity of quantum states and facilitating fault-tolerant quantum computation [6]. In recent years, significant advancements have been made in the field of QEC, leading to the development of several robust codes [7,8,9,10]. These codes employ principles from classical error correction, such as error detection and syndrome measurement, and adapt them to the unique properties of quantum systems. One of the most widely used frameworks for QEC is the stabilizer formalism [11,12], which encompasses popular codes like the Steane [13], Shor [14], and surface codes [15,16]. These codes provide varying degrees of safety against different types of errors, such as bit flips, phase flips, and more complex errors. To explore and simulate these error correction codes, Qiskit [17], an open-source quantum computing framework developed by IBM, offers a comprehensive set of tools. It provides a set of tools and libraries for programming, simulating, and executing q-algorithms using different backends, including real quantum hardware. In Qiskit, there are built-in functionalities and libraries specifically designed for quantum error correction. These tools provide developers with the necessary resources to study, implement, and experiment with different error correction codes. Qiskit's support for quantum error correction empowers researchers and practitioners to build robust quantum algorithms and applications that can overcome the inherent fragility of q-systems. Overall, Qiskit and QEC are interconnected concepts that play a vital part in advancing the field of q-computing [18,19]. With Qiskit, users can construct and manipulate quantum circuits, simulate the effects of noise and decoherence, and apply various error correction codes to protect quantum information.

Qiskit's high-level language allows for efficient coding and easy integration of quantum error correction techniques, making it an invaluable resource for studying and implementing QEC algorithms. In conclusion, QEC is a very much needed field in q-computing that addresses the challenges posed by noise and decoherence. Through the use of sophisticated error correction codes and simulation frameworks like Qiskit, researchers and developers can explore and enhance the resilience of quantum computations. By leveraging these tools, we can pave the way for more reliable and scalable quantum technologies that will revolutionize various domains, from cryptography and optimization to drug discovery and materials science.

2. Computational Procedures

2.1 Qubits

The primary unit of information in the classical information system is known as a bit, which can be either 1 or 0. If there are n number of bits, then the system possess 2^n possible states which are represented through a vector of dimension n over Z_2 . On the contrary, qubits serve as the foundational unit of information in q-information systems. The basis vectors are

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

An unknown and arbitrary qubit $|\psi\rangle$ can be written as,

$$|\psi\rangle = a_0|0\rangle + a_1|1\rangle = a_0 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + a_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}, \text{ which is a superposition state of two orthonormal}$$

eigenstate $|0\rangle$ and $|1\rangle$ with the complex coefficients a_0 and a_1 such that $|a_0|^2 + |a_1|^2 = 1$.

A different orthonormal basis known as the Hadamard basis, denoted by $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

and $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ is also useful for various other computations.

2.2 Quantum Register

In a classical system, a register contains a sequence of bits, whereas in a quantum system, a register contains a sequence of qubits. The state of the qubits in a quantum register is described by a complex vector in a high-dimensional space known as Hilbert space. The number of qubits in a register decide the size of the Hilbert space and thus the computational power of a quantum system. Quantum registers are essential components for implementing quantum algorithms and performing quantum computations. Operations such as quantum gates and measurements are applied to the qubits within a quantum register to manipulate and extract information from the quantum state. The entanglement of qubits within a register enables the exploration of quantum phenomena like quantum parallelism and quantum entanglement, which are the basis for many quantum algorithms and protocols.

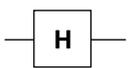
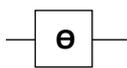
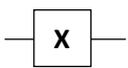
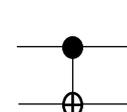
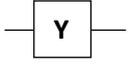
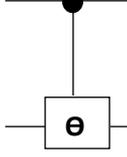
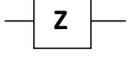
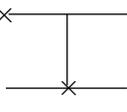
The vector $|\psi\rangle$ with n qubits state of the quantum register can be defined as,

$$|\psi\rangle = \sum_{k=0}^{2^n-1} s_k |a\rangle = a_0 \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + a_1 \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} + \dots + a_{2^n-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

which contains a superposition of 2^n basis states, where complex numbers s_k must fulfil the condition $|\psi\rangle = \sum_{k=0}^{2^n-1} |s_k|^2 = 1$.

2.3 Q-Gates

Table-1:

Gate	Gate Name	Matrix	Symbol	Name	Matrix
	Hadamard-H Gate	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$		Rotation Gate	$\begin{bmatrix} 0 & 1 \\ 1 & e^{i\theta} \end{bmatrix}$
	Pauli-X Gate	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$		Controlled Not Gate	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$
	Pauli-Y Gate	$\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$		Controlled Rotation Gate	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-j\theta} \end{bmatrix}$
	Pauli-Z Gate	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$		Swap Gate	$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

2.4 The Q- CNOT Gate

The quantum CNOT (Q-CNOT) gate, also represented as the controlled-NOT gate or CX gate, is a fundamental two-qubit quantum logic gate in q-computing. It operates on two qubits, usually one qubit is referred to as the control qubit (C) and the other qubit is referred to as target qubit (T). The Q-CNOT gate operates a NOT operation on the target qubit (T) whenever the control qubit (C) is in the q-state $|1\rangle$. If the control qubit (C) is in the q-state $|0\rangle$, then the target qubit (T) remains to be unaffected. Mathematically, the Q-CNOT gate can be represented as $CNOT|c\rangle|t\rangle = |c\rangle|c \otimes t\rangle$, where the ket vector $|c\rangle$ denotes the control

qubit, $|t\rangle$ denotes the target qubit, and \otimes denotes the XOR (exclusive OR) operation. In other words, Q-CNOT gate reverses the q-state of the target qubit if and only if the control qubit is in the q-state $|1\rangle$.

2.5 Depolarizing Channel

A depolarizing channel is a type of noise channel that can introduce errors in q-systems. In QEC, the goal is to safeguard q-information from the detrimental effects of noise and errors that naturally occur during quantum computations or quantum communication. The depolarizing channel is a commonly studied and modeled noise channel. A depolarizing channel acts on a quantum state by applying either the identity operation (with probability $1 - p$) or a completely random Pauli operation with equal probabilities. The parameter p represents the probability of an error occurring on each qubit. Mathematically, the action of the depolarizing channel on a quantum state ρ is given by:

$$D(\rho) = (1 - p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z)$$

where X , Y , and Z are the Pauli matrices and ρX , ρY , and ρZ are the states obtained by applying the respective Pauli operations to ρ . The depolarizing channel is called "depolarizing" because it tends to decrease the coherence and entanglement of the input state, effectively "depolarizing" it towards a maximally mixed state.

2.6 Logical Error Rate

In the scheme of QECCs, the logical error rate gives the probability of an error occurring in the logical information encoded in a quantum state, after error correction procedures have been applied. QECCs are designed to safeguard the q-information from the effect of errors introduced during quantum computation or transmission. They achieve this by encoding the information redundantly across a larger number of physical qubits. These codes have the ability to detect and reverse errors, thereby preserving the encoded information. When QEC is performed, errors affecting the physical qubits can still occur. These errors could be due to various sources, such as environmental noise or imperfect operations. However, the main aim of error correction is to mitigate these errors and reduce their impact on the logical information. The LER quantifies the residual errors that affect the logical qubits after error correction. It represents the probability that a logical error, which cannot be corrected by the error correction code, occurs per logical qubit per gate operation. In other words, it measures how often an error affects the logical information despite the application of error correction. Reducing the LER is crucial in quantum computation, as it directly affects the reliability and accuracy of quantum algorithms and calculations. The development of more efficient and robust error correction codes, as well as advancements in error mitigation techniques, aim to minimize the LER and improve the overall performance of quantum systems.

3. Mathematical background of various QECCs

QECCs involve the encoding of k qubits into a subspace of the Hilbert space consisting of n qubits. To ensure accurate decoding, it is necessary that at most t qubits are influenced by errors in the encoded state of the qubits. These codes are represented by the notation $[[n, k, d]]$,

where 'd' gives the code's distance and enables the encoding of k logical qubits into n physical qubits. Numerous latest QECCs have been discovered [4,13,20,21]. Additionally, researchers have successfully implemented quantum error correction through various experiments [22,23,24]. However, it is important to note that simulation of QECCs have not been extensively explored by many researchers, despite its significance in the development of quantum computers. Building quantum circuits remains a particularly challenging task. One notable advancement is the formation and execution of a three-qubit quantum computer using a quantum simulator. Quantum computer simulators play a crucial role as optimal quantum hardware architectures continue to emerge. While qubit circuit simulation takes precedence, qubit gates also offer their own advantages. Reducing the number of gate count and circuit levels contributes to minimizing the occurring errors and total costs in quantum circuits.

Table 2: Some quantum error correcting codes

Code	n	k	Error				
			X	Z	Y	H	X, Z
Bit flip	3	1	✓	✗	✗	✗	✗
Phase flip	3	1	✗	✓	✗	✗	✗
Five qubit	5	1	✓	✓	✓	✓	✗
Steane	7	1	✓	✓	✓	✓	✓
Shor	9	1	✓	✓	✓	✓	✓

3.1 Three-Qubit Bit Flip Code

The bit flip code for 3 qubits is a QECC that safeguard against errors (bit-flip) on a single qubit [25]. It is an easiest demonstration of an error correcting code designed to preserve the information which is accumulated in qubits and reduce the impact of noise and errors in q-computing systems. In this code, three qubits are utilized to encode a single logical qubit. The logical qubit represents the information that needs to be protected. The encoding process involves applying specific quantum gates to the three physical qubits to create an entangled state. The process of encoding in this code can be represented as follows: $|0\rangle_L$ (logical qubit) = $|000\rangle_p$ (physical qubits), $|1\rangle_L$ (logical qubit) = $|111\rangle_p$ (physical qubits). Here, $|0\rangle_L$ and $|1\rangle_L$ denote the logical states $|000\rangle_p$ and $|111\rangle_p$ and represent the physical states of the three qubits. To correct for errors, this code uses the concept of majority voting. After applying quantum gates and performing operations on the encoded qubits, measurements are made to determine the presence of any errors. If an error is detected, a majority vote is taken among the three physical qubits to identify the correct logical value. For example, if an error appears and flips a physical qubit, the majority vote would determine the correct logical value based on the majority of the three qubits. The incorrect qubit would be flipped back to the correct state, restoring the original information. Overall, the code provides a way to safeguard against a

single occurrence of bit-flip error, ensuring the integrity of the logical qubit in a q-computing system.

The easiest quantum channel of the three-qubit bit flip code used to encode a single qubit using ancillary qubits is given in figure-3.1.1. From figure-3.1.3, we observe that the LER obtained over depolarizing channel with parameter 0.4 increases for the increased range of the error probability. All qubits are initialized in the $|0\rangle$ or $|1\rangle$ state for each computation.

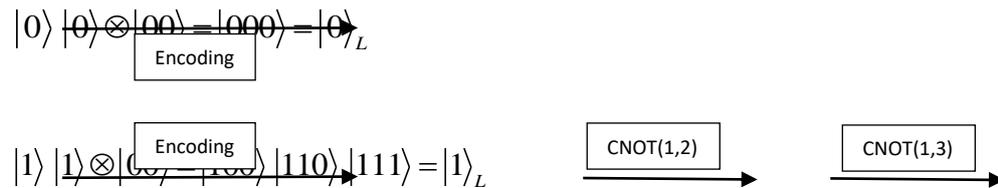


Figure-3.1.1: Quantum channel for bit-flip code

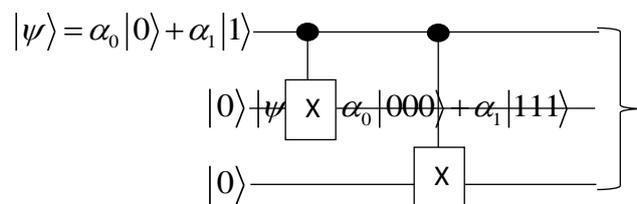


Figure-3.1.2: Quantum circuit for bit-flip code

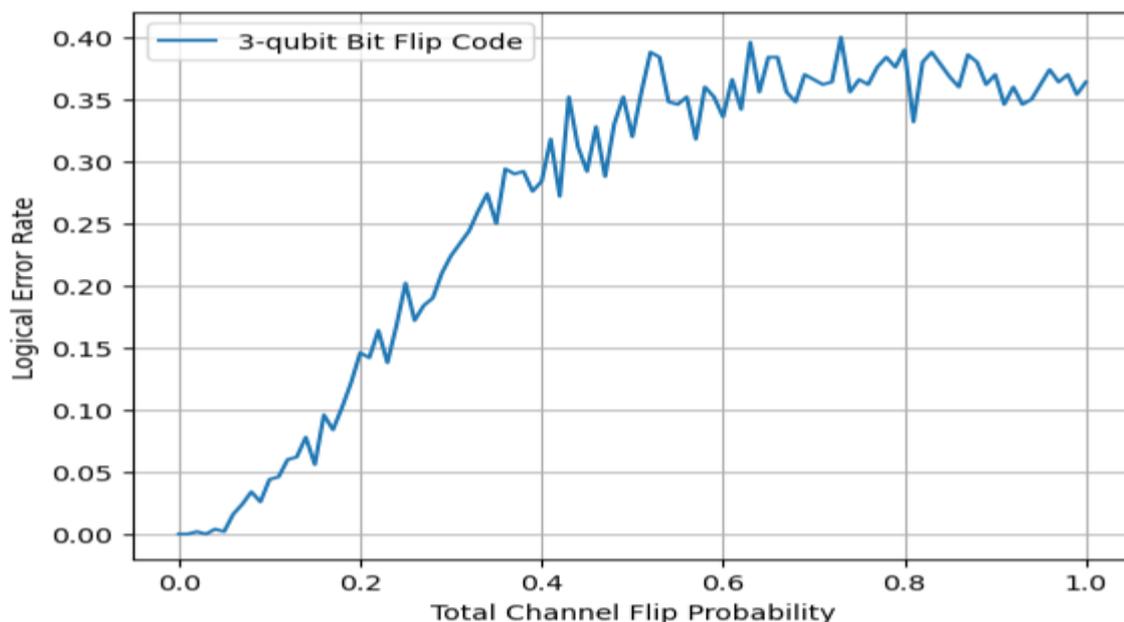


Figure-3.1.3: Performance of three-qubit bit flip code

3.2 Three Qubit Phase Flip Code

The three-qubit phase flip code is another QECC that protects against phase-flip errors on a single qubit [26]. Similar to the three-qubit bit-flip code, it is designed to preserve the

information stored in qubits and lower the influence of noise and errors in q-computing systems. In the three-qubit phase flip code, three qubits are utilized to encode a single logical qubit. The encoding process involves applying specific quantum gates to the three physical qubits to create an entangled state. The encoding process of this code can be represented as follows: $|0\rangle_L$ (logical qubit) = $|+++ \rangle_P$ (physical qubits), $|1\rangle_L$ (logical qubit) = $|--- \rangle_P$ (physical qubits). Here, $|0\rangle_L$ and $|1\rangle_L$ stand for the logical states, and, $|+++ \rangle_P$ and $|--- \rangle_P$ represent the physical states of the three qubits. To correct for errors, the 3-qubit phase flip code utilizes a combination of measurements and quantum gates. After performing operations on the encoded qubits, measurements are made to determine the presence of any errors. If an error is detected, specific quantum gates are executed to the physical qubits to correct the error and restore the original information. For illustration, if an error flips the phase of one of the physical qubits, the corresponding measurement will detect the error. Then, the appropriate quantum gate will be operated to the qubits to reverse the error and restore the logical state. The three-qubit phase flip code provides a way to protect a single phase-flip errors, ensuring the integrity of the logical qubit in a quantum computing system. By utilizing entanglement and error correction techniques, quantum systems can mitigate the impact of noise and errors, improving the overall reliability of quantum computations. From figure-3.2.2, we observe that the LER obtained over depolarizing channel increases for the increased range of the error probability just as in the case of the above code.

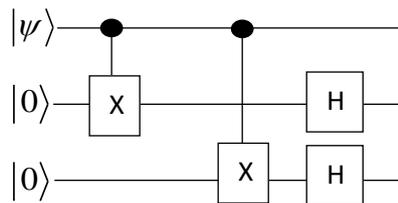


Figure-3.2.1:Quantum phase flip code circuit

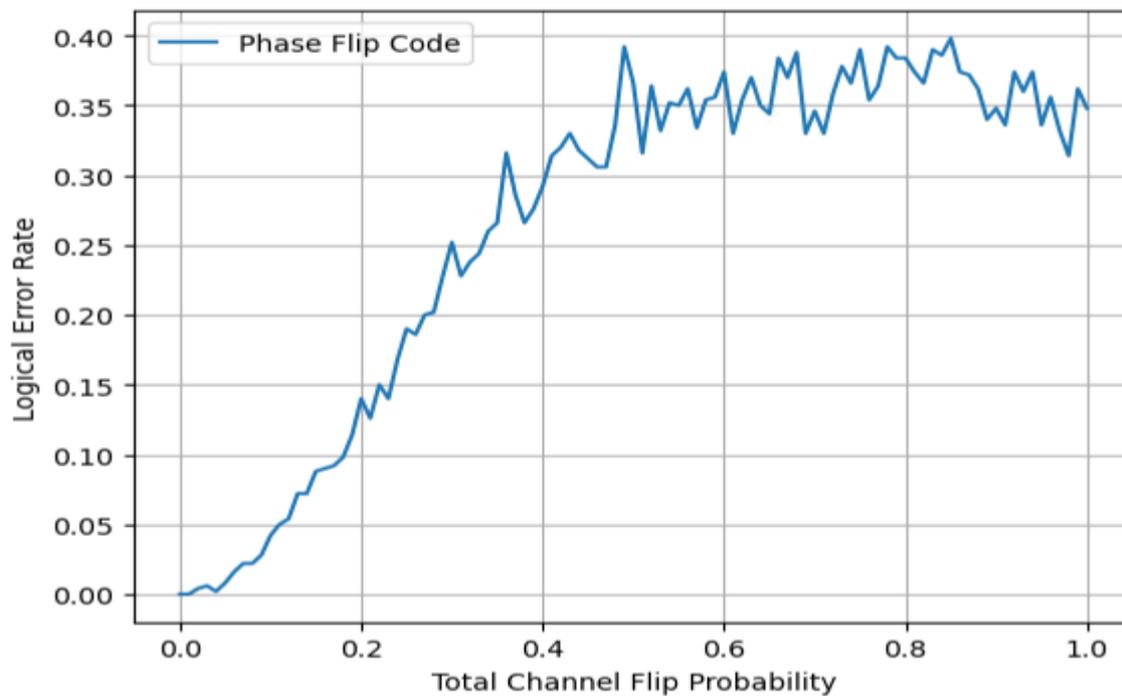


Figure-3.2.2: Performance of three-qubit phase flip code

3.3 Five Qubit Code

The 5-qubit error correction code, commonly known as $[[5,1,3]]$, is a QECC designed to safeguard a single logical qubit against single-qubit errors such as bit-flip and phase-flip errors [21]. It belongs to the class of stabilizer codes and is extensively employed as a fundamental component in various QEC strategies. In the 5-qubit error correction code, a group of five physical qubits is utilized to encode one logical qubit. The encoding process involves applying specific quantum gates to the physical qubits to create an entangled state. The encoding process of the 5-qubit error correction code can be represented as follows: $|0\rangle_L$ (logical qubit) = $(|00000\rangle_L + |11111\rangle_L)/\sqrt{2}$ and $|1\rangle_L$ (logical qubit) = $(|00011\rangle_L + |11100\rangle_L)/\sqrt{2}$. Here, $|0\rangle_L$ and $|1\rangle_L$ represent the logical states, and the corresponding expressions indicate the entangled states of the 5 physical qubits. To correct errors, the 5-qubit error correction code utilizes the notion of stabilizer measurements. Stabilizer operators are used to spot and correct errors in the encoded state. The stabilizer measurements involve measuring specific combinations of the physical qubits and comparing the results with the expected values. By performing measurements and comparing the outcomes to the expected values, it is achievable to identify and correct errors. If an error is detected, appropriate operations can be implemented to the physical qubits to take back the original logical state. The 5-qubit error correction code provides a way to protect against single-qubit errors, enhancing the robustness of quantum computations. It forms the basis for more advanced error correction codes and QEC schemes, allowing for the detection and correction of errors that arise due to noise and imperfections in quantum computing systems. From figure-3.3.1, we see that the LER tends to increase for the increasing values of the error probability.

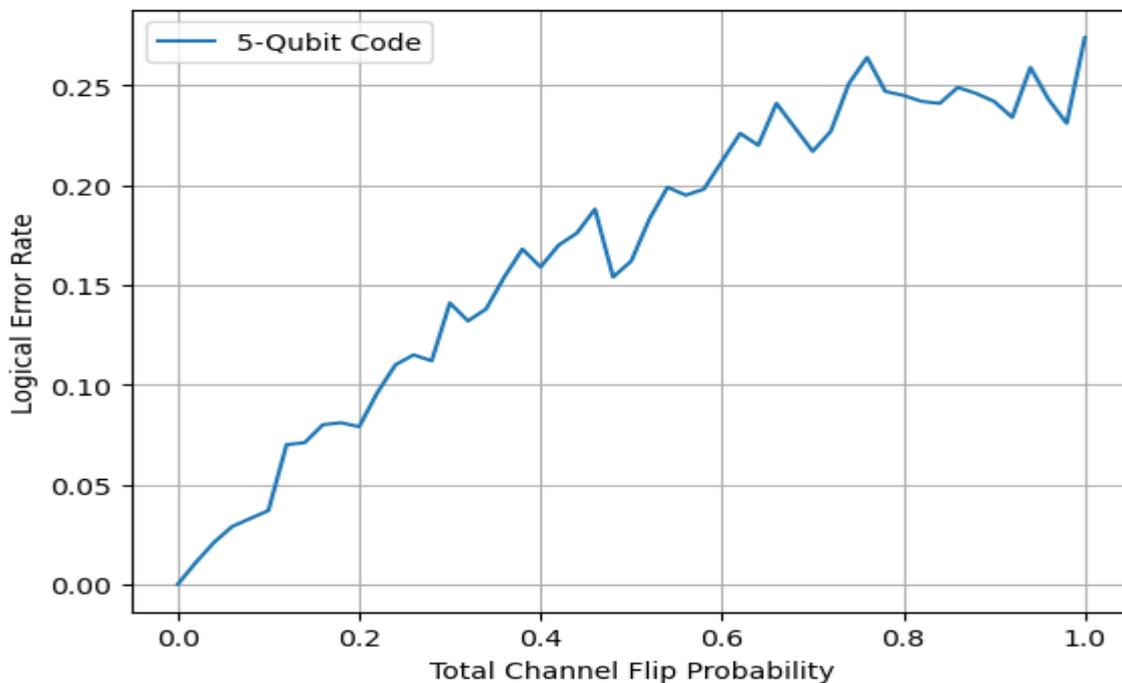


Figure-3.3.1: Performance of five qubit code

3.4 Steane Code

The Steane code [13] is a QECC that provides protection against errors in quantum computations. It is named after its creator, Andrew Steane, who introduced it in 1996. This code is a sample of a stabilizer code, specifically a single-qubit error-correcting code. The Steane code is designed to correct single-qubit errors, including bit-flip (X) and phase-flip (Z) errors, as well as their combinations. It encodes a single logical qubit into seven physical qubits, offering a higher level of error protection. The encoding process of the Steane code involves creating a sizeable number of physical qubits and utilizing a combination of q-gates to encode the logical qubit. The specific encoding is achieved by applying a set of Hadamard (H), phase (S), CNOT (CX), and Toffoli (CCX) gates to the physical qubits. The Steane code employs stabilizer measurements, similar to other stabilizer codes, to spot and correct errors. The stabilizer measurements involve applying a collection of stabilizer operators, derived from the encoded states and gate operations, to the physical qubits. By performing measurements on the stabilizer operators and comparing the outcomes to the expected values, errors can be identified and corrected. If an error is detected, appropriate gates are operated to the physical qubits to restore the original logical state. The Steane code is particularly known for its ability to correct arbitrary single-qubit errors using a combination of measurements and gate operations. It is widely used in quantum error correction schemes and forms an essential building block in fault-tolerant quantum computing. By utilizing the Steane code, quantum computing systems can mitigate the resulting impact of noise and errors, enhancing the accuracy and reliability of q-computations. Its application contributes to the evolution of more robust quantum computers which are capable of executing complex calculations with improved error correction capabilities. From figure-3.4.1, we see that the LER is increasing for the increasing values of the error probability.

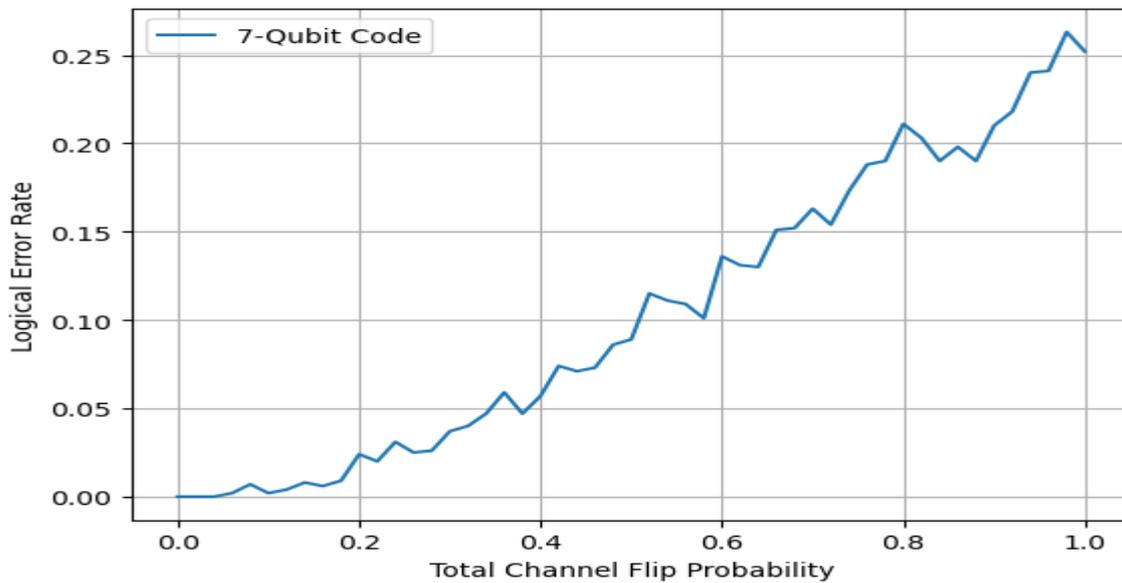


Figure-3.4.1: Performance of Steane code

3.5 Shor Code

The Shor code is a QECC that provides protection against errors in quantum computations [1]. It is specifically designed to correct errors introduced by faulty gate operations and environmental noise in quantum systems. The Shor code is an specimen of a stabilizer code and is named after mathematician Peter Shor, who introduced it in 1995. The Shor code is a powerful error correction code that can correct arbitrary errors on multiple qubits. It is primarily used to protect quantum information during long computations and maintain the integrity of the encoded logical qubits. The encoding process of the Shor code involves creating a broad number of physical qubits to encode a smaller number of logical qubits. The specific encoding depends on the desired level of error protection. The code utilizes a combination of various q-gates such as Hadamard gates (H), CNOT gates (CX) and phase gates (S), to create entanglement and encode the logical qubits into the physical qubits. From figure-3.5.2, we see that the LER increases for the increased values of the error probability.

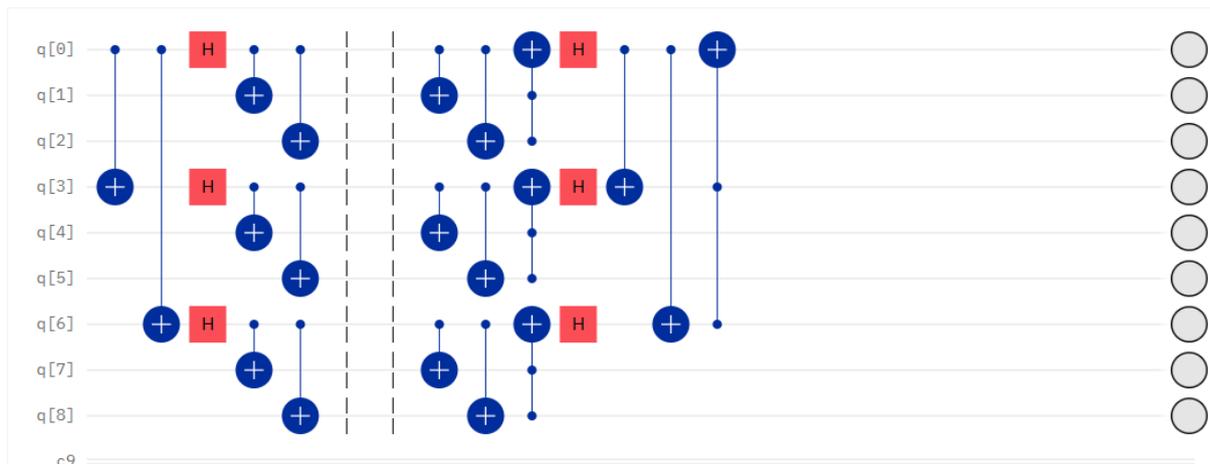


Figure-3.5.1:Quantum Shor code circuit

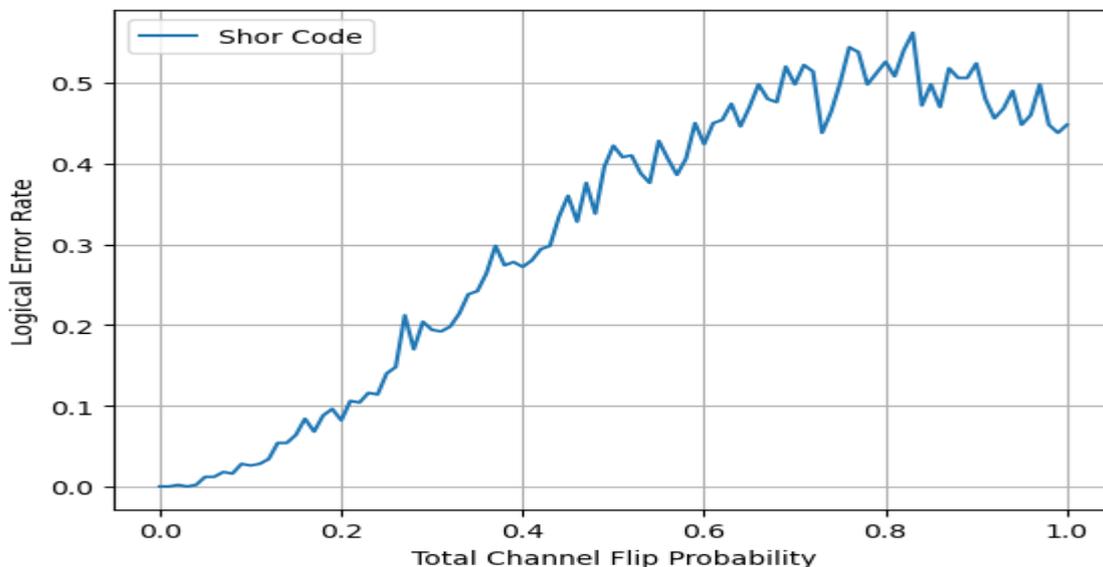


Figure-3.5.2: Performance of Steane code

4. Implementation Work using Qiskit

The main feature of the implementation method is to originate the encoding and decoding Qiskit programs given the information of the QECCs in terms of its circuit and its error correcting capabilities. Our initial state is either $|0\rangle$ or $|1\rangle$ with equal probability. The principle reason for this is that since we only measure in the Z basis, it is impossible to correctly identify how many times the procedure for error correction has failed if the initial states are different from the computational basis states. We consider a depolarizing noise model with varying depolarizing probability. We then plot LER vs total channel flip probability.

a) Selection of Error Correcting Codes: Begin by choosing the error correcting codes to be implemented in Qiskit. Consider codes, for instance, the three-qubit bit flip code, phase-flip code, five-qubit code, Steane code and Shor code, which are commonly used and well-

studied. These codes exhibit different characteristics and trade-offs, providing insights into their performance under channel flip errors.

b) Quantum Circuit Construction: Utilize the Qiskit framework to develop the error correction circuits for the selected codes. Define the required number of physical qubits, logical qubits, and ancillary qubits based on the specific error correcting code being implemented. Qiskit provides a comprehensive set of tools and functions to facilitate circuit construction.

c) Error Model: Define the error model that includes the channel flip error as the primary source of noise. The channel flip probability represents the likelihood of a qubit experiencing a bit flip error during a quantum computation. Set a range of channel flip probabilities to be investigated, spanning from low noise to high noise regimes.

d) Error Introduction: Incorporate the channel flip errors into the quantum circuits by randomly applying bit flip operations to the physical qubits with the defined probabilities. This step simulates the presence of noise in the quantum system.

e) Syndrome Measurement: Implement the necessary error syndrome measurement operations based on the selected error correcting code. Measure the stabilizer generators or parity checks to determine the presence and locations of errors in the system.

f) Detection and Correction of Errors: Utilize standard techniques for error detection and correction to identify and correct errors based on the measured syndromes. Apply appropriate recovery operations to restore the logical qubits to their desired states.

g) Statistical Analysis: Repeat the simulation for each channel flip probability multiple times to collect sufficient data for statistical analysis. Calculate the LER by counting the number of errors remaining after error correction and normalization by the number of encoded logical qubits.

h) Data Collection: Record the LER obtained for each channel flip probability and error correcting code combination. Create a dataset that captures the relationship between LER and total channel flip probability for analysis and visualization.

i) Graphical Representation: Construct a graph that depicts the LER on the y-axis and the total channel flip probability on the x-axis. Plot the data points for each error correcting code to visualize the performance of different codes in mitigating errors under varying noise conditions.

j) Analysis and Comparison: Analyze the graph to draw conclusions about the effectiveness of the error correcting codes in reducing logical errors. Compare the performance of different codes, taking into account factors such as code distance, circuit depth, and error correction capabilities. Identify regimes where specific codes outperform others and gain insights into the strengths and limitations of each code.

k) Discussion and Interpretation: Discuss the results obtained, providing explanations for observed trends and trade-offs between LER and total channel flip probabilities. Interpret the findings in the context of quantum error correction and its implications for developing reliable quantum computing systems.

5. Result Analysis and Comparative Explications Graph

The simulation of QECCs in Qiskit for analyzing the LER versus total channel flip probability provides valuable insights into the performance of different codes under varying

noise conditions. The analysis of the collected data allows for insight of the robustness and effectiveness of each error correcting code. Here, we present the key findings and observations resulting from the implementation and analysis.

Comparison of Error Correcting Codes:

Logical Error Rates: The LER for each code are plotted against the total channel flip probability. This analysis reveals how different codes respond to increasing noise levels.

Code Distance and Performance: Codes with higher code distances generally exhibit lower LER, indicating improved error correction capabilities. However, codes with longer distances often require more resources and increased circuit depths, which can impact practical implementations.

Optimal Noise Regimes: Certain codes may outperform others in specific noise regimes. For example, a code with a shorter code distance may be more effective in mitigating errors in a noise regime characterized by predominantly low channel flip probabilities.

Trade-offs between Code Distance and Circuit Depth:

Circuit Depth: As the code distance increases, the circuit depth also tends to increase. This trade-off needs to be considered when choosing an error correcting code, as deeper circuits may be more challenging to implement in real-world quantum systems.

Resource Constraints: Practical considerations, such as limited qubit resources and gate fidelities, may impact the choice of error correcting codes. Balancing code distance and circuit depth becomes crucial to optimize error correction capabilities while accommodating resource limitations.

Practical Implications:

Selection of Error Correcting Codes: The results aid in selecting appropriate error correcting codes based on the specific noise characteristics and resource constraints of a quantum system. The analysis provides guidance for choosing codes that strike a balance between error correction capabilities and practical implementation considerations.

Designing Robust Quantum Computing Systems: Understanding the relationship between LER and channel flip probabilities is essential for designing reliable and fault-tolerant quantum computing systems. The results can inform strategies for error detection, correction, and fault tolerance, enhancing the overall system performance.

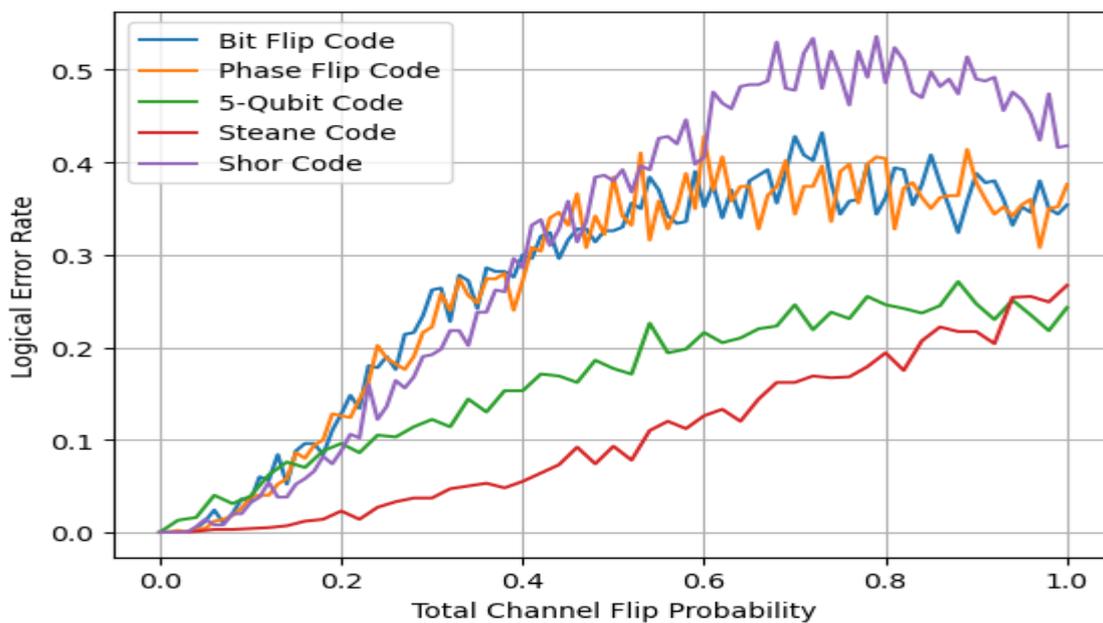


Fig.-5.1: Comparative performance of bit-flip, phase-flip, five qubit, Steane and Shor codes for 1000 shots

Comparative performance of ‘three-qubit bit flip code, phase flip code, five qubit code, Steane code and Shor code’ are depicted in Fig.-5.1 under depolarizing channel with probability $p = 0.4$. Observing the ranking of the "best" code in a given noise model reveals that it is dependent not only on the noise model itself but also on the magnitude of the noise present. This phenomenon becomes evident as logical error rates for different codes intersect, presenting a somewhat unexpected result. Particularly, this effect is noticeable in experiments involving bit flip and phase flip noise. In the case of bit flip noise, it is observed that the bit flip code outperforms the phase flip code and the Steane code (both exhibiting similar performance) up to a bit flip probability of 0.5. However, beyond this threshold, the situation completely reverses. It is noteworthy that in practical scenarios where the logical error rate surpasses 0.5, as demonstrated in the bit flip noise plot, it is advisable to apply a logical X gate to encoded qubits—a strategy akin to classical approaches for Binary Symmetric Channels with error probabilities exceeding 0.5.

5. Conclusion

Our results demonstrate the varying effectiveness of different error correcting codes in mitigating errors and preserving logical qubit states. We observe that codes with higher code distances generally exhibit lower LER, but at the expense of increased circuit depth. Additionally, we identify regimes in which certain codes outperform others, highlighting the importance of selecting appropriate error correcting codes based on the specific noise characteristics of the quantum system. Overall, this study provides valuable insights into QEC on Qiskit, offering a comparative analysis of different codes for error correcting and their performance in the presence of depolarizing channel flip errors. The findings contribute to the ongoing research and development of robust quantum computing systems by guiding the selection and optimization of error correction strategies.

6. Futuristic orientation

- **Extension to Other Noise Models:** The analysis can be expanded to examine the performance of error correcting codes under different noise models, such as amplitude damping noise or coherent errors, to get a more comprehensive interpretation of their error correction capabilities.
- **Optimization of Code Implementations:** Further research can focus on optimizing the implementation of error correcting codes to reduce circuit depth, resource requirements, and increase fault tolerance.
- **Experimental Validation:** The results obtained from simulations can be validated through experiments on real quantum hardware to assess the potency of error correction codes in practical scenarios.

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